BĪRŪNĪ'S *BOOK OF PEARLS CONCERNING THE PROJECTION OF SPHERES*

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Introduction

This paper presents the Arabic text, a translation, and a commentary upon a treatise by Abū al-Rayḥān al-Bīrūnī, the preeminent eleventh century scientist of Central Asia[1].

Bīrūnī's writings ranged over a broad spectrum of topics which included among others astronomy, mathematics, geography, astrology, metallurgy, and pharmacy. Only a few of these works have been studied. He also wrote several works on the astrolabe[2] - an instrument used to solve more than three hundred standard mathematical astronomical problems[3] - and only one of these has been properly studied[4]. These works deal with different aspects of the instrument: some are detailed and include a detailed description of the theory behind the instrument[5] as well as the

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[1] For Bīrūnī's biography see [7].

[2] Boilot, [5], Nos. 46–50, pp. 191–193; Sezgin [12], vol. 6, pp. 268 seqq., Nos. 6, 7, 8, 9, 10, 11, 14, 19, 28.

[3] For an introduction to the theory, construction, and use of the astrolabe, see [13], [6], [9], and [8].

[4] For the only work of Bīrūnī which deals with projections and which has been published, see [3] and [11].

[5] The basic theory used in the construction of an astrolabe is that of stereographic projection. In this method the pole of projection is taken on the surface of the celestial sphere, and the plane of projection is the plane perpendicular to the diameter of the sphere passing through this point. Two properties of this kind of projection are:

1. All circles on the sphere, that do not pass through the pole of projection, project as circles on the plane of projection; thus circles are preserved.

2. The mapping is conformal, that is, an angle subtended by two curves on the sphere is equal to the angle subtended by the projections of these curves on the plane of projection; thus angles are preserved.

The proof for the first property of this projection was known in antiquity

manner of its operation, while others deal with the practical aspects involved in its construction.

The treatise studied here is called: *Kitāb al-Durar fī saṭḥ al-ukar* [hereafter *Durar*] ("The Book of Pearls Concerning the Projection of Spheres")[6], and is, to use Bīrūnī's own words, a work on the "making of the astrolabe . . .", and "the method of determining the horoscope with the various kinds of astrolabes". The treatise is composed of two parts called "questions". The first is in eight sections which cover the theoretical and the practical problems of constructing the instruments, and treats the following topics:

The first section considers the problem of mapping the celestial sphere on a plane by projecting each point of the sphere onto the equatorial plane from a fixed point on the north-south axis of the sphere. The author examines the different kinds of projections that result from various positions of the pole of projection along the axis of the sphere. The pole of projection could lie either inside or outside the sphere, or on either one of its two poles, and the projections would vary accordingly. Bīrūnī also maintains in this section that the circles of the sphere that are projected on an astrolabe could be either great circles, the horizon and azimuth circles, or small circles, these being the day circles or the almucantars. The projections of the above circles could be straight lines, circles, parabolas, hyperbolas, or ellipses.

In section two, Bīrūnī starts by saying that for purposes of astrolabic projections, the pole of projection is taken to coincide with either one of the two poles of the sphere; the resulting projection would then be what is now called stereographic. He goes on to determine the method for constructing the day circles in northerly and southerly astrolabes. Section three deals with the

(see Apollonius, in [2], pp. 1–14), while the second part was not proved until much later. For more details and proofs on this theory, and its relationship to astrolabes, see [8], pp. 27–29.

[6] For information on available copies of this manuscript, see Sezgin [12], vol. 5, p. 381, no. 4, vol. 6, pp. 269–270. Note that this text is different from *Tasṭīḥ al-ṣuwar wa-tabṭīḥ al-kuwar*, which was edited by A. Sa'īdān in [11], and of which [3] is an English translation by Berggren. This treatise although listed in [5] under the works on the astrolabe, has no direct mention of the actual practical construction or usage of that instrument. It is related to astrolabes only as far as general projection theories are concerned.

method for constructing horizon circles, and although those are given for northerly astrolabes, it is indicated that those are symmetrical with southerly ones.

Section four discusses the projection of almucantars of altitude and of depression, and section five discusses the projection of azimuth circles. In section six, the method of dividing the ecliptic into zodiacal signs is presented after repeating the method of constructing the map of the ecliptic circle, which is presented earlier, at the end of section three. Then in section seven the method for determining or rather constructing the pointers for the stars is presented. Finally section eight gives the method of obtaining the hour lines.

The second "question" is in six sections. It deals with the problem of horoscope determination for the planispheric astrolabes discussed in the first question, as well as for a number of astrolabes that Bīrūnī mentions elsewhere in his works[7].

The Manuscripts

Three copies of the *Durar* were used to establish the text below, all of them from the Bodleian Library of Oxford[8]. These manuscripts are:

1. Seld. Sup. [3297,85], fol. 1–10v. This is used as the base manuscript for the accompanying edition[9].
2. Thurston [3970,3], fol. 120r–122r[10].
3. Marsh 713, fol. 239v–242v[11].

Seld. Sup. 85 is used as the base manuscript because the name of its scribe was specified, in addition to its writing date. The manuscript clearly states that it was composed by al-Bīrūnī. Moreover, the additions on the margins of this manuscript indi-

[7] Such problems are also discussed by Ṣūfī, for example, in [13], chapters 51–64.

[8] The author wishes to acknowledge his gratitude to the Keeper of the Oriental Collection at the Bodleian for his cooperation in supplying these manuscripts for this study.

[9] See [15], pp. 226–227.

[10] *Ibid*, p. 198. The manuscript also contains a collection of several works by Bīrūnī and others.

[11] *Ibid.*, p. 203–204.

cate that it was compared with the original text from which it was copied, and that scribal errors were corrected. Seld. 85 is thus a reliable copy.

Thurston 3 is part of a *Majmū'* containing mathematical and astronomical treatises. Herein, al-Bīrūnī's text is rendered in its complete form. Since the second "question" does not appear in the Seld. Sup. manuscript, we had to depend completely on the Thurston and Marsh manuscripts in editing this part.

Manuscript Marsh 713 is believed to be a recent copy of Thurston 3, and therefore was of little use for our purpose. In fact, at times, the readings of the copyist of Marsh 713 were so bad that they actually distorted the text of Thurston 3 rather than elucidated it. After a close study of Marsh 713, and after much consideration, it was thought that the copyist of that manuscript did not understand what he was copying, and that the manuscript ought to be used with extreme caution, seeking its reading only when the other two were totally illegible. Even then, one had to guess at the meaning intended in the text, and accept it only if it made sense in the context. Luckily this did not have to be done very often, and the edited text, as it now stands, is probably the closest approximation to the one that was originally intended by Bīrūnī.

A number of the figures appearing in Seld. Sup. 85 were distorted, and they were redrawn in accordance with the text. As for the drawings in the other manuscripts, they were not used, either because the diagrams did not appear to start with, or because such diagrams were fully distorted. Figures 13 through 17 are not part of the Arabic text and were added in the English translation for illustration purposes.

The Authorship

The text of the *Durar* is mentioned in Boilot's index among the works of Bīrūnī[12], but not under the works on the astrolabe. In the manuscript copies used for this edition, the authorship could be directly assertained only once, namely in the case of the first manuscript where the name of Bīrūnī is mentioned on the

[12] See [5], pp. 225, No. 143.

flyleaf. But the same manuscript did not have the name repeated in the body of the text as would have been the usual practice.

Internal evidence, however, leaves no doubt that this work was written by Bīrūnī, for there were several indications that pointed to him as the only possible author. One such instance, for example, resulted from a detailed comparison between the contents of the *Durar* with the contents of another treatise unquestionably written by Bīrūnī, namely his elaborate work called *Istī'āb al-wujūh al-mumkinah fī ṣan'at al-asṭurlāb*. One of the many similarities that can be traced in the two works is what Bīrūnī refers to as the method of al-Khujandī for the construction of azimuth circles[13], which was the method preferred by Bīrūnī in both treatises.

A second instance is the observation used to determine the value of the maximum inclination of the ecliptic as being 23; 35 degrees. In the *Durar* Bīrūnī gives the diameter of the ring used in the observation as being 15 cubits, and the year in which the observation was conducted as the year 385 of the Hijra. These figures happen to be the same figures given by Bīrūnī in another one of his securely attributed books, namely the *Kitāb Taḥdīd nihāyāt al-amākin*[14].

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The author would like to express his gratitude to Prof. E. S. Kennedy, who originally suggested this study and supplied copies of the manuscripts, to Prof. F. Sezgin for making available microfilms of two of the three manuscripts, and to Prof. M. Kashef who helped in the translation of the two Persian verses of poetry that were added on the flyleaf of manuscript No. 1.

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[13] See translation below.

[14] See Jamil Ali's translation of the *Taḥdīd*, [1], p. 77.

[ب ١ و] كتاب الدرر في سطح الأكر

تأليف الشيخ الأجل العالم العامل أبي ريحان محمد بن أحمد البيروني
غفر الله له ورحمه وعفى [عنـ]ـه[١]

[ب ١ ظ] بسم الله الرحمن الرحيم وبه نستعين على الظالمين. الحمد لله رب العالمين وصلى الله على محمد خاتم النبيين وعلى آله وصحبه أجمعين[٢]. كنت حررت لمولاي[٣] ما خطر بالبال وتصور في الوقت والحال من أعمال تسطيح الأكر وتصويرها بما أمكن من أنواع الصور، حتى ظننت وأظن[٤] أني آستوعبت جميع ما يمكن أن يتوهم فيه، ونبهت على حقائقه بعض التنبيه، وإن كنت جردتها عن البراهين إذ كنت مأموراً بتحصيل[٥] القوانين على الوجوه الصناعية والطرق المثالية.

والآن لما أمرني أيده الله[٦] بإفراد عمل يخص صنعة الأصطرلاب[٧] دون غيره، والإنباء عن كيفية آستخراج الطالع في سائر أنواع الأصطرلابات التي عاينتها[٨] أو أشار إليها ليكون ذلك منبهاً[٩] على كيفية سائر الأعمال فيه. وأنا ممتثل ما أمره[١٠] مرتسماً رسمه مستعيناً بالله وهو حسبي ونعم الوكيل[١١].

[١] ورقة العنوان سقطت من ت. – [عنـ]ـه: غير واضحة في ب وثبتناها للمعنى. – [٢] بسم الله ... اجمعين: سقطت في ت وورد مكانها: بسم الله الرحمن الرحيم، ربي أنعمت فزد. – [٣] لمولاي: سقطت من ت وورد مكانها: لك. – [٤] وأظن: أو أظن في ت. – [٥] مأموراً بتحصيل: مسؤولاً حينئذٍ عن تحصيل في ت. – [٦] لما أمرني أيده الله: سقطت في ت وورد مكانها: لما توقعت آبتلاء الأيام الجائرة عن سننها وعود هلالي (مكاني) مزين جمع ذوي الألفة والمحبة، سألني إفراد ... – [٧] ت: المستعمل. – [٨] ب: عينها، وهي تهجئة قرآنية ل عاينها. – [٩] منبهاً: مبيناً في ب وآخترنا منبهاً للمعنى. – [١٠] ممتثل ما أمره: فاعل ما شانه في ت. – [١١] الوكيل: المعين في ت.

*Translation**

[f. 1r] *The Book of Pearls Concerning the Projection of Spheres.* Written by the Honorable Scholar, al-Shaikh abī al-Rayḥān Muḥammad b. Aḥmad al-Bīrūnī, May God Forgive and Exonerate Him.

[f. 1v] In the name of God, the Merciful (and) the Compassionate, whose aid we seek against oppressors. Praise be to God the Lord of Creation. And may His prayers be upon Muḥammad the seal of the Prophets, and upon all his family and companions.

I had composed for my lord concerning the projections of spheres and their representation with whatever figures possible, that which had occurred to my mind, and that which I (could) imagine at that time and (under those) conditions. I also assumed, and still do, that I had already covered all that could be thought of. I also drew attention to some of the basic principles (of projections), without mentioning any of the proofs, for I was asked to deduce the laws from the practical and theoretical aspects[15].

And now that he, may God support him, has asked me to devote a special work to the making of the astrolabe alone, and to report about the method of the determination of the horoscope in all the various kinds of astrolabes that I myself had observed, or that he himself had indicated – in order for that to be a reminder of the various workings of it (i. e. the astrolabe) – I am hereby responding to his request and following his command, applying to God for aid, for He is my refuge and He is the best of guardians.

* References in square brackets are to the folio numbers in the base manuscript Seld. Sup. 85, and words in parenthesis are not part of the Arabic text, but are added to clarify the translation.

[15] Bīrūnī is possibly referring in this paragraph to the elaborate treatise that he wrote on the astrolabe, called *Istīʿāb al-wujūh al-mumkinah fī ṣanʿat al-asṭurlāb*. The book matches the description in that it covers many basic construction principles, without giving proofs, to be used as a practical reference on the subject. If this is so, then Bīrūnī is adressing his speech, when he says my lord, to the same person, who is specified in *al-Istīʿāb* as Abū Sahl ʿĪsā b. Yaḥyā al-Ṭabarī, otherwise unknown.

[ب ٢ و] السؤال الأول: وهو ثمانية فصول [١٢]

القول الأول: في كيفية تشكيل ما في الكرة على السطوح بآختلاف قطب التسطيح على المحور.

التسطيحات بالمخروطات تقع من المراكز[١٣]

١ - إما إلى جهة القطب الشمالي:

أ - إما على المحور داخل الكرة: (١) دوائر. (٢) وخطوط مستقيمة. (٣) وأنواع القطوع الثلاثة.

ب - وإما على القطب نفسه: (١) دوائر. (٢) وخطوط مستقيمة.

ج - وإما على المحور خارج الكرة: (١) دوائر. (٢) وقطوع نواقص[١٤]. (٣) وخطوط مستقيمة.

٢ - أو إلى جهة القطب الجنوبي:

أ - إما على المحور داخل الكرة: (١) دوائر. (٢) وخطوط مستقيمة. (٣) وأنواع القطوع الثلاثة.

ب - وإما على القطب نفسه. (١) دوائر. (٢) وخطوط مستقيمة.

ج - وإما على المحور خارج الكرة: (١) دوائر. (٢) وقطوع نواقص. (٣) وخطوط مستقيمة.

والدوائر المتشكلة في الأصطرلاب:

١ - إما عظام وهي إما

أ - آفاق: [ب ٢ ظ] (أ) الكرة المنتصبة: خطوط مستقيمة متقاطعة على مركز الصفيحة، لا يمكن غير ذلك.

[١٢] فصول: - أقاويل في ت. - أعيد ترتيب شكل المواد الواردة في القول الأول مع الحفاظ على نفس المعلومات، وذلك للتبسيط وقد كتب هذا القول على شكل جدول في ب وت، مع آختلاف في شكل كل من الجدولين. - [١٣] المراكز: مركز العالم في ت. - [١٤] قطوع: وردت خطأً قوع في ب.

[f. 2r] THE FIRST QUESTION: And it consists of eight sections.

Section one: On the method of representing (projections) of what is (found) on the sphere on planes, according to the variation of (the position) of the pole of projection on the axis (of the sphere).

Conic projections (vary in accordance with the position of the pole of projection) with respect to the centers (of the spheres).

1. (If the pole of projection) were in the direction of the north pole, (then it) would be:

a. Either on the axis inside the sphere, (and then the conic projections would be): Circles, straight lines, and (one of) the three conic sections.

b. Or (it would be) on the (north) pole itself, (and then the conic projections would be): Circles and straight lines.

c. Or (it would be) on the axis outside the sphere, (and the conic projections would then be): Circles, hyperbolas, and straight lines.

2. (If on the other hand) it were in the direction of the south pole, (then it would be):

a. Either on the axis inside the sphere, (and thus the conic projections would be): Circles, straight lines, and (one of) the three conic sections.

b. Or (it would be) on the (south) pole itself, (and thus the conic projections would be): Circles and straight lines.

c. Or (it would be) on the axis outside the sphere, (and the conic projections would thus be): Circles, hyperbolas, and straight lines.

(As for) the circles that are represented on the astrolabe, they are:

1. Either great circles and these are:

a. Horizon (circles): (In the case of) the right sphere they would be straight lines [f. 2v] intersecting at the center of the

(ب) أو الكرة المائلة: (١) دوائر في التسطيح على القطبين. (٢) وأنواع القطوع الثلاثة إذا زال عنها.

ب – وسموت: (١) + دوائر في التسطيح على القطبين[١٥]. + (٢) + وأنواع القطوع الثلاثة إذا زال عنها[١٦]. +

٢ – وإما صغار وهي إما:

أ – مدارات، شمالية وجنوبية: دوائر لا يمكن غير ذلك. إلا أنها تتبادل في القطبين. وتختلف أوضاعها + بٱختلاف موضع التسطيح[١٧]. +

ب – أو مقنطرات: ٱرتفاعية وٱنحطاطية: يمكن أن تشكلها دوائر إذا كان. التسطيح على القطب نفسه. ويمكن أن تكون من أنواع القطوع الثلاثة إذا زال عنها القطب، داخلاً أوخارجاً. وتقع منها خطوط مستقيمة على التسطيحات الجنوبية.

القول الثاني: في تخطيط المدارات.

ولكن لما قصدنا في هذا الموضع العمل على القطبين نفسيهما[١٨]، فإنّا نقول أنّ المدارات تتسطح دوائر متوازية. وذلك أنّا نفرض الصفيحة دائرة ا ب ج د على مركز ه وقطري ا ه ج، ب ه د[١٩]. ونأخذ بقدر الميل الأعظم وهو على ما وجدناه بالرصد بخوارزم في سنة خمس وثمانين وثلثمائة للهجرة[٢٠] بدائرة [ب ٣ و] قطرها خمسة عشرة ذراعاً، ثلاثة وعشرون جزءاً[٢١] وثلث وربع.

ونصل ب ز يقطع قطر ا ه ج على نقطة م ونجعل نقطة ه مركزاً وندير ببعد ه م دائرة م ط ک فيكون معدل النهار وهو مدار الحمل والميزان وفيه ثلاث نقط في

[١٥] + دوائر ... القطبين +: سقطت من ب. – [١٦] + وأنواع ... عنها +: سقطت من ب.

[١٧] + بٱختلاف ... التسطيح +: غير واضحة في ت أو م. – [١٨] نفسيهما: نفسهما في ب.

[١٩] ب ه د: غير مقروءة في ب، غير واضحة في ت وخطأ في م. وقد أثبتت بالرجوع إلى الرسم.

[٢٠] للهجرة: للهجرية في ب. – [٢١] جزءاً: أسقطت الهمزة في ب و ت وأضيفت هنا حسب الهجاء الحديث.

plate, and nothing else could be possible. In the case of oblique spheres they would be circles, if the projection is from the two poles, or the three conic sections if it were otherwise.

b. Azimuth circles: (And they would project as) circles when the projection is from the two poles, or the three conic sections if it were otherwise.

2. Or small circles, and they are either:

a. Day circles – to the north or to the south – and they would (project as) circles and nothing else would be possible. However, they could interchange (positions) with respect to either one of the two poles, and their positions would vary with the variation of the position (of the pole) of projection.

b. Or almucantars – in altitude or in depression – and they could be represented by circles if the projection (pole) is on the pole itself, or they could be one of the three conic sections if the projection (pole) were removed from the poles either internally or externally. Some of these would project as straight lines in the southerly projections.

Section two: On the construction of the day circles.

Since we intend at this point to carry (the projection) from the two poles themselves, we would then say that the day circles would be projected as concentric circles. Let us then assume the plate to be circle *ABGD* (Figs. 1, 2), with center *E*, and diameters *AEG* and *BED*. We take (arc *AZ* to be equal to) the inclination (ε) of the ecliptic, which was found to be twenty-three degrees and a third and a fourth ($23;35^0$), according to the observation that we ourselves conducted in Khwārizm in the year three hundred and eighty-five **[f. 3r]** of the Hijra (= A. D. 994/5) by using an (observational) ring with a diameter equal to fifteen cubits (*circa* 9 meters).

We join *BZ*, and let it cut the diameter *AEG* at point *M*. With point *E* as center, and radius *EM*, we draw circle *MTK*, which

تربيعات وهي م ط ك. وأما نقطة م فمنها مأخذ[٢٢] الميل إلى كل واحدةٍ من جهتيها: الشمالي إلى جهة ك، والجنوبي إلى جهة ط . فأما نقطة[٢٣] ط فقطب التسطيح للأصطرلاب الشمالي. والوصل يقع بينه وبين ما يظهر لنا من سائر النقط ٱلتي نستخرجها للعمل.

ونريد أن نحفظ ما نحتاج إليه[٢٤] أن يكون غير مؤثر للٱستغناء عنه عند تمام العمل بالحمرة، وٱلتي[٢٥] نحتاج أن تكون مؤثرة بالسواد. فليفهم الناظر ذلك[٢٦]. وأما[٢٧] نقطة ك فقطب التسطيح للأصطرلاب[٢٨] الجنوبي، ومنه يقع الوصل بينه وبينها.

ثم نمثل أولاً لمدار شمالي [ب ٣ ظ] فنأخذ مقدار ميله من نقطة مأخذ الميل إلى جهة ك. وليكن قوس م ح. فإن كنا نعمل لأصطرلاب شمالي وصلنا ط ح، يقطع[٢٩] ا ه ج على س. فيكون ه س نصف قطر ذلك المدار على مركز ه. وإن كنا نعمل لأصطرلاب جنوبي وصلنا ك ح وأخرجناه على ٱستقامته[٣٠] حتى يلقى[٣١] ا ه ج على ل. فيكون ه ل نصف قطره على مركز ه.

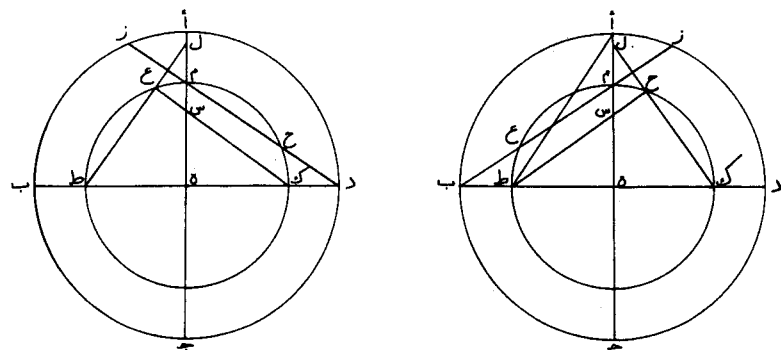
ثم نمثل لمدار[٣٢] جنوبي ونأخذ ميله من نقطة مأخذ الميل إلى جهة ط. وليكن م ع. ففي الأصطرلاب الشمالي نصل ط ع ل[٣٣] مستقيماً. فيكون ل ه نصف قطره على مركز ه. وفي الجنوبي نصل ك س ع فيكون س ه نصف قطره على مركز ه. وعند تساوي الميلين[٣٤] في جهتين مختلفتين يظهر تبادل المدارين في نوعي الأصطرلاب. فقد ظهرت[٣٥] كيفية عمل المدارات في الصفيحة وذلك ما أردنا أن نبين.

[٢٢] مأخذ: مدخل في ب، وقد تكون خطأ من الناسخ لأن البيروني يستعمل «مأخذ» لاحقاً في النص.
[٢٣] نقطة: نقط في ب. — [٢٤] إليه: سقطت في ت. — [٢٥] وٱلتي: او ٱلتي في ت.
[٢٦] ذلك: سقطت في ت. — [٢٧] ت: على. — [٢٨] للأصطرلاب: سقطت في ت.
[٢٩] يقطع: بقطر في ت. — [٣٠] ٱستقامته: ٱستقام في ت. — [٣١] يلقى: نلق في ب.
[٣٢] لمدار: المدار في ب، ولمداري في ت. — [٣٣] ط ع ل: ط ف ل في ت وأخذنا بقراءة ب لٱنسجامها مع الأشكال. — [٣٤] الميلين: المثلثين في ب. — [٣٥] ظهرت: تكررت في ت.

would then be the equator, which is the day circle of (the beginning) of either Aries or Libra. (This equator) would have the three points *M*, *T*, and *K* at quadratures. Point *M* would be the point from which the declination is measured in either direction: northward towards *K*, and southward towards *T*. As for point *T*, it would be the pole of projection for a northerly astrolabe, and (all lines issuing from it) would connect it with all points apparent to us, which are used in the projection.

(Construction) lines that we would not need in the final projection for work (necessities), would be drawn in red. Others would be drawn in black. Let any one examining it note that. As for point *K* it is the pole of projection of the southerly astrolabe, and from it (will issue) all lines that connect it to the points (to be projected).

We first take the example of a northerly day circle (Fig. 1). [f. **3v**] We measure the amount of its inclination from the reference point of inclination (i. e. *M*), in the direction of point *K*. Let that be the arc *MH*. If we are constructing a northerly astrolabe, we join *TH*, which will cut *AEG* at *S*. Thus *ES* is the radius of that day circle around center *E*. But if the astrolabe under construction were southerly, then we would join *KH* and extend it straight, to intersect *AEG* at point *L*. Thus *EL* is the radius (of that day circle) around center *E*.

We next take an example of a southerly day circle (Fig. 2), and measure the amount of its inclination from the inclination reference point, in the direction of point *T*. Let that be arc *MO*. In the northerly astrolabe we join the straight line *TOL*. *LE* would then be the radius (of that day circle) around center *E*. In the southerly one we join *KSO*, to obtain *SE* as a radius (of the day circle) around center *E*. When the two inclinations are equal in opposite directions, the interchangeability between the two day circles, in the two kinds of astrolabes, would be obvious. Therefore, the method of constructing the day circles on the plate is now clear, and that is what we wanted to show.

شكل ١

شكل ٢

[ب ٤ و] القول الثالث[٣٦]: على الآفاق:

وأما آفاق الكرة المنتصبة فإنها متشكلة خطوطاً مستقيمة وليس يقع منها في الصفيحة إلاخط نصف النهار والليل و[٣٧] خط المشرق والمغرب. وظهورهما يغني عن الإرشاد إليهما. وأما سائر الآفاق فإنا نعيد لها الصفيحة بقطريها ومدار الحمل. فنقول[٣٨]: أن التقسيم يوجب أن نجعل من الآفاق نصيباً للشمال ونصيباً للجنوب. لكنه لما كان النصف الجنوبي مجهول الكيفية غير معلوم الحالة عندنا[٣٩]، آستغنينا عنه عند[٤٠] ذكر الآفاق للعروض الجنوبية وقصرنا الهم على[٤١] الشمالية، وإن لم يكن الفضل بينهما سوى التبديل والخلاف في جهة أخذ[٤٢] عرض البلد.

ثم نقول أنا إذا أردنا تخطيط أفق لعرض معلوم[٤٣]، أخذنا من نقطتي طَ، كَ قوسين، كل واحدة منهما بمقدار العرض المفروض. [ب ٤ ظ] أما من طَ فإلى جهة مَ كقوس طَزَ. وأما من نقطة كَ فإلى خلاف تلك الجهة، وهو[٤٤] كَحَ. وسمينا نقطتي زَحَ طرفي[٤٥] الافق.

[٣٦] الثالث: الثاني في ب. – [٣٧] و: وفي في م وغير واضحة في ت. – [٣٨] فنقول: ونقول في ت. – [٣٩] عندنا: سقطت من ت. – [٤٠] عند: عنه في جميع المخطوطات والأفضل قراءة عند. – [٤١] على: عن في ب. – [٤٢] أخذ: كتبت فوق السطر في ب. – [٤٣] معلوم: مفروض في ت. – [٤٤] هو: هي في ت. – [٤٥] طرفي: وردت خطأً طرفا في ت، وأضيف بعدها كلمة قطر.

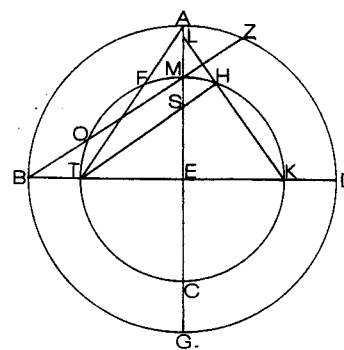



Fig. 1

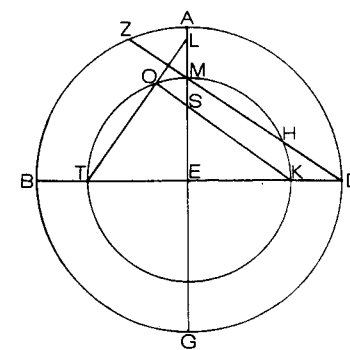



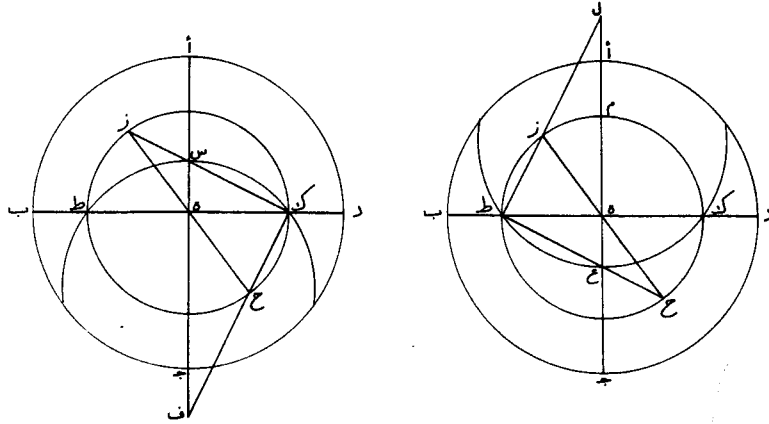
Fig. 2

[f. 4r] *Section three*: On the horizon (circles).

As for the horizons of the right sphere ($\varphi = 0^0$) they would project in straight lines, and none of them will be shown on the plane except the meridian line and the east-west line. Since they are obvious we do not need to discuss them any further. For the remaining horizon circles we redraw the plate (of the astrolabe) together with its two diameters, and the equator circle. We then say: It is necessary, according to the division (of the globe), that we take some of the horizons to be northerly while others to be southerly. But since the conditions of the southern half (of the sphere) are unknown to us, we will designate them when we consider the horizon circles of the southerly latitude, and we will limit our concern to the northerly ones, in spite of the fact that the only difference between the two is the interchangeability of the direction of measurement of the geographical latitude.

We then say that if we wanted to project the horizon for a given latitude, we measure from points *T* and *K* (Figs. 3, 4) two arcs, each of them being equal to the value of the assumed latitude. **[f. 4v]** The (arc taken) from point *T* would be in the direction of *M*, such as arc *TZ*, while the one from point *K* would be in the opposite direction; let that be *KH*. We thus call the two points *Z* and *H* the two extremities of the horizon.

فإن كان الأصطرلاب شمالياً وصلنا ط ز ل، ط ع ح فيكون ل ع قطر الأفق في الصفيحة وعلى وضعه، ويكون مركز الأفق في منتصف[٤٦] ما بين نقطتي ل، ع. وإن كان جنوبياً وصلنا ك س ز، ك ح ف. فيكون س ف قطر الأفق وعلى وضعه، ويكون مركزه في منتصف ما بين نقطتي ف، س[٤٧]. ثم ندير على مركز الافق وببعد نصف قطره ما وقع منه على الصفيحة. وعلامة صحته أن يقاطع مدار الحمل والميزان[٤٨] على نقطتي ط ك لاغيره، فهذا هو عمل الآفاق في الصفائح.

وبمثله نخط في العنكبوت منطقة البروج إذا فرضنا فيه عرض الإفق، أعني كل واحدة من قوسي ط ز، ك ح بمقدار تمام الميل الأعظم وهو معلوم[٤٩]. وذلك ما أردنا بيانه.

شكل ٤ شكل ٣

[٤٦] منتصف: سقطت في ت. — [٤٧] ف، س: س، ف في ت. — [٤٨] والميزان: أسقطت في ت. — [٤٩] معلوم: سقطت في ت وأضيف مكانها: ست وستون جزءاً وربع وسدس جزء.

If the astrolabe were northerly, we join *TZL* and *TOH* (Fig. 3). *LO* would then be the diameter of the (projected) horizon on the plate (of the astrolabe), as indicated. And the center of the (projected) horizon would be the midpoint of *LO*. (However), if it (i. e. the astrolabe) were southerly, we join *KSZ* and *KHF* (Fig. 4). *SF* would then be the diameter of the (projected) horizon, as indicated. Its center would also be the midpoint of *FS*. Then with the center of the (projected) horizon, and with a radius equal to the horizon's radius, we draw the portion of the circle that would appear on the plate. The indication (that the projection) is accurate is that it would intersect the day circle of Aries and Libra (i. e. the equator) at the points *T* and *K* only. That is how horizons are projected on plates.

In a similar manner we project the ecliptic on the rete, by assuming it to have the latitude of the horizon – I mean that each of the two arcs *TZ* and *KH*, would be equal to the complement of the maximum declination, which is also known. And that is what we wanted to show.

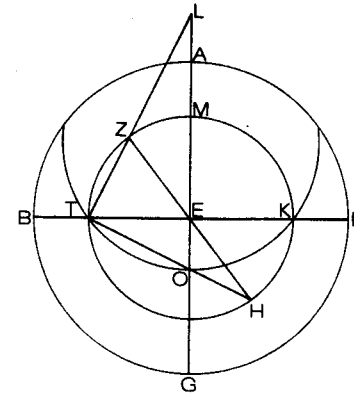
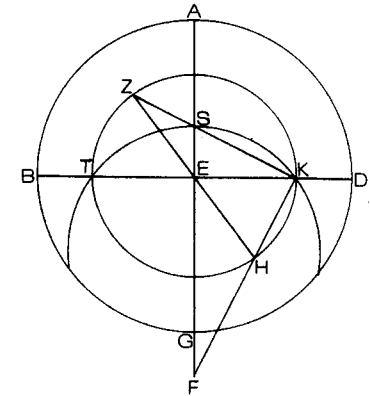



Fig. 3 Fig. 4

[ب ٥ و] القول الرابع: على المقنطرات.

وأما المقنطرات فعلى[٥٠] نوعين، أحدها الموازية[٥١] للأفق فوق الأرض وتسمى مقنطرات الآرتفاع. والأخرى الموازية لها تحتها وتسمى مقنطرات الآنحطاط. فأما آلتي للآرتفاعات فلا بد منها في كل الأصطرلابات[٥٢]. وأما آلتي للآنحطاطات فيحتاج[٥٣] إليها في الأصطرلابات المركبة من الأجناس المختلفة، وربما آحتيج في الأصطرلاب المستعمل إلى بعضها، أو[٥٤] آستعين عليها لآستخراج[٥٥] شيء من الأعمال.

ونعيد لعملها[٥٦] الصفيحة بقطريها ومدار الحمل وطرفي قطر الأفق. ونقول أولاً: أن نحن[٥٧] إذا أردنا تخطيط مقنطرة[٥٨] في الأفق آلذي طرفا قطره نقطتا زح فإنا نأخذ من لدن نقطتي ح، ز. أما إذا كانت للآرتفاعات، فإلى جهة سمت الرأس من كل واحدة منها[٥٩]، قوساً [ب ٥ ظ] بقدر[٦٠] آرتفاع تلك المقنطرة المفروضة كقوسي زل، حع في الصورة الأولى. وأما إذا كانت للآنحطاطات فإلى[٦١] جهة سمت الرجل[٦٢] من كل واحدة منهما قوس بقدر آنحطاط تلك المقنطرة كقوسي زل، حع من[٦٣] الصورة الثانية. ثم نصل في الشمالي طلس، طفع[٦٤]. فيكون سف[٦٥] قطر تلك المقنطرة، وعلى مركزها[٦٦] منتصف ما بين نقطتي سف. وفي الجنوبي نصل كعن وكصل[٦٧]. فتكون صن قطر تلك المقنطرة وعلى وضعه[٦٨]، ومركزها في منتصف ما بين نقطتي صن. ونديرها على مركزها، وببعد نصف قطرها. فهذا عمل المقنطرات

[٥٠] فعلى: فهي في ت. — [٥١] الموازية: موازية في ت. — [٥٢] الأصطرلابات: أسطرلاب في ت. — [٥٣] فيحتاج: فنحتاج في ت. — [٥٤] أو: و في ت. — [٥٥] لآستخراج: بآستخراج في ت. — [٥٦] لعملها: صححت في هامش ب وكانت لهما. — [٥٧] أن نحن: سقطت من ت.
[٥٨] مقنطرة: مقنطرات في ت. — [٥٩] منها: منهما في ت. — [٦٠] بقدر: بمقدار في ت.
[٦١] فإلى: إلى في ت. — [٦٢] ت: نأخذ. — [٦٣] من: في، في ت.
[٦٤] طفع: طعح في ب، وغير مقروءة في ت. وفضلنا طفع للمضمون، ولآنسجامها مع الشكل. — [٦٥] سف: غير مقروءة في ت. — [٦٦] وعلى مركزها: غير مقروءة في ت.
[٦٧] كصل: سقطت من ب وت وأضفناها لإتمام المعنى في النص. — [٦٨] نصل ... وضعه: غير مقروءة في ت.

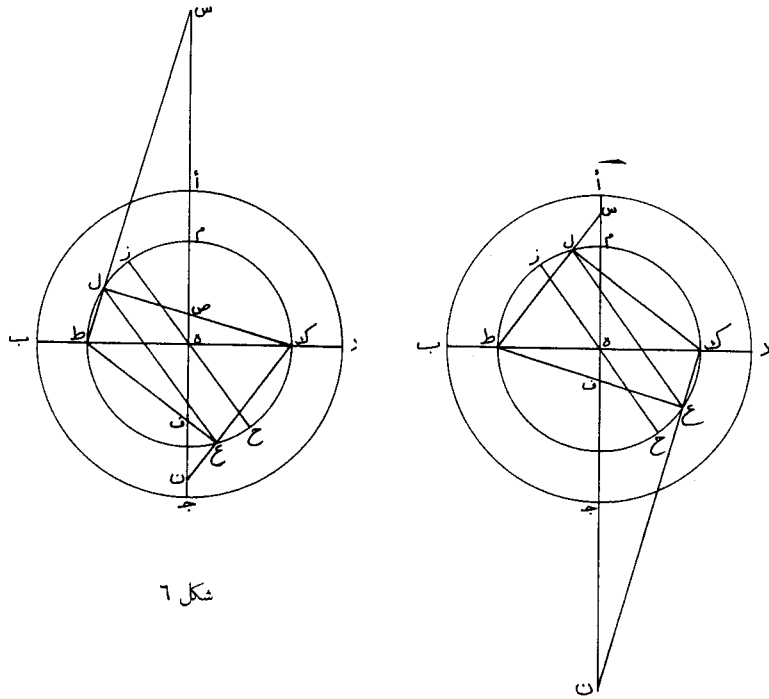
[f. 5r] *Section four*: On the almucantars.

As for the almucantars, they are of two kinds: Some are parallel to the horizon in the visible part of the firmament and are called almucantars of altitude, while some are below the horizon and parallel to it – and these are called the almucantars of depression. As for the (almucantars) of altitude, they are needed in all kinds of astrolabes, while those of depression are only needed in astrolabes that combine various types, and some of them may also be needed in the astrolabes commonly used; or others could still be used to aid in some of the constructions.

To map (these almucantars) we repeat (the construction of) the plate, with its two diameters, the day circle of Aries, and the two extremities of the horizon's diameter. We first say: If we want to project the almucantar of the horizon which has the two points Z and H as the extremities of its diameter, we would then take from points H and Z, for the altitude ones, an arc **[f. 5v]** equal to the altitude of the given almucantar in the direction of the zenith, such as arcs ZL and HO in the first diagram (Fig. 5). While for those belonging to the depression we would take, in the direction of the nadir, from each of the two points, arcs equal to the amount of depression of that almucantar, such as arcs ZL and HO in the second diagram (Fig. 6). Then we join, in the northerly (astrolabe), TLS and TFO. Thus SF would be the diameter of that almucantar, and its center would be the endpoint of SF. While in the southerly one we join KON and KCL. Thus CN would be the diameter of the almucantar as indicated. Its center is the midpoint of CN. (The almucantar) could then be drawn using this center and a radius equal to half the diameter (CN). This is (the operation) of mapping the engraved almucantars for elevation and depression. Anyone drawing a comparison

المخطوطة للاّرتفاع والاّنحطاط. ولا يخفى على المماثل أنها لعرض واحد تتبادل في نوعي الأصطرلاب في جهتي سمت الرأس والرجل. وذلك ما أردنا أن نبين.

شكل ٦

شكل ٥

ولمقنطرات [ب٦و] الاّنحطاطات للاّصطرلاب الشمالي، آلتي هي مقنطرات الارتفاعات[٦٩] في الجنوبي، خواص يغنينا[٧٠] إعواز العمل إليها عن[٧١] ذكرها، إلا واحدة[٧٢]. ونعيد لها الصفيحة ونقول: أنا متى أخذنا مقدار آنحطاط المقنطرة المفروضة في الأصطرلاب الشمالي من طرفي الأفق فآنتهينا من عند حَ إلى لَ، ومن عند زَ[٧٣]

[٦٩] الاّرتفاعات: الاّرتفاع في ت. – [٧٠] يغنينا: تعنينا في ب. – [٧١] عن: من في ت.
[٧٢] واحدة: واحد في ت. – [٧٣] زَ: لَ في ت.

between these two kinds of astrolabes will find it obvious that for the same latitude, (the almucantar) could be interchanged in the direction of the zenith, or in the direction of the nadir. And this is what we wanted to show.

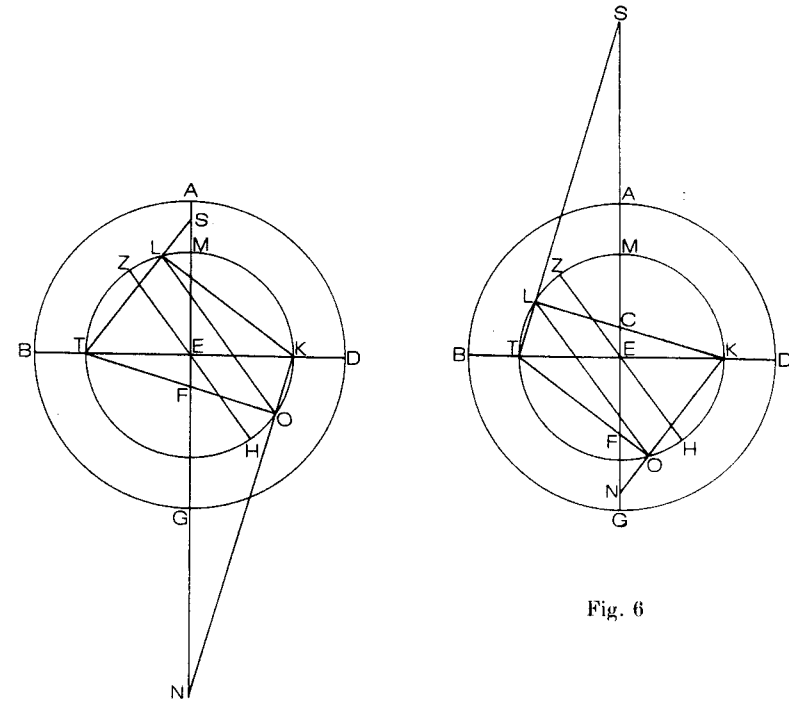
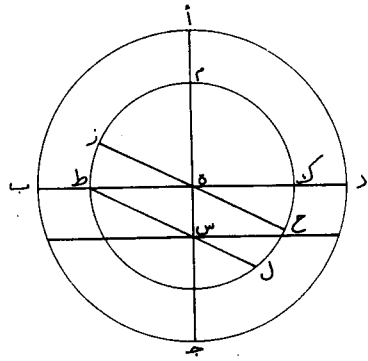
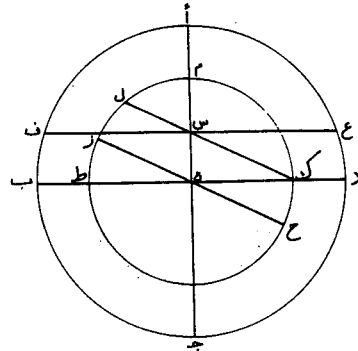



Fig. 5

Fig. 6

The depression almucantars **[f. 6r]** of a northerly astrolabe, which are the altitude almucantars for a southerly one, have properties which we need not mention, because we do not use them, except for one. For that one we redraw the plate and say that after we take the amount of depression of the assumed almucantar in the northerly astrolabe, from the two extremities of the horizon, such that (in Fig. 8) we reach from H to L and from

إلى ط نفسه وصلنا ط س ل كما في الصورة الأولى. ثم أخرجنا على س خط ع س ف مستقيماً موازياً لقطر ب ه د. فتكون تلك المقنطرة. وفي الآرتفاعات في الأصطرلاب الجنوبي متى أخذنا آرتفاع المقنطرة المفروضة من طرفي الأفق[٧٤] فانتهينا[٧٥] من عند ز إلى ل ومن عند ح إلى ك نفسه وصلنا ك س ل وعملنا ما تقدم ذكره، كما في الصورة الثانية. فيكون ع س ف تلك المقنطرة، وذلك ما أردنا.

شكل ٨　　　　شكل ٧

[ب ٦ ظ]　القول الخامس: على السموت.

وفي عمل دوائر[٧٦] السموت طرق كثيرة ذكرنا بعضها في ذلك الكتاب وسنستعمل الآن طريقة أبي محمود الخجندي[٧٧] لخفتها وسهولتها. ونعيد الصفيحة بمدار الأفق والحمل[٧٨]. ونأخذ قوس م ز بقدر عرض البلد، ونصل في الصورة الأولى ط ز يقطع

[٧٤] الأفق: قطر الأفق في ت. — [٧٥] ب: إلى، أسقطت لآستقامة النص كما آستعمله البيروني سابقاً: (فآنتهينا من عند ح إلى ل ومن عند ز إلى ط.) — [٧٦] دوائر: صححت على الهامش في ب وكانت مسقطة. — [٧٧] أبي محمود الخجندي: أبي حامد محمود بن الخجندي في ت. — [٧٨] الأفق والحمل: الحمل والأفق في ت.

Z to *T* itself; then we join *TSL* as in the first diagram (Fig. 5). We then let the straight line *OSF* be issued from point *S* parallel to the diameter *BED*. This will thus be the almucantar. As for altitude (almucantars) in the southerly astrolabe, we take the altitude of the assumed almucantar, from the two extremities of the horizon, such that (in Fig. 7) we reach from *Z* to *L* and from *H* to *K* itself. We then join *KSL*, and proceed in the second diagram as we have done before. *OSF* would thus be the (desired) almucantar. And that is what we wanted to show.

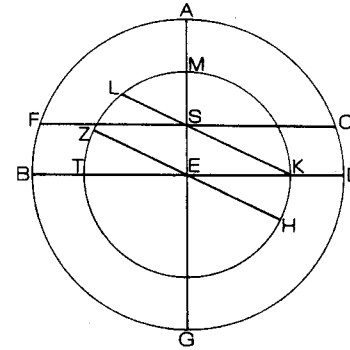
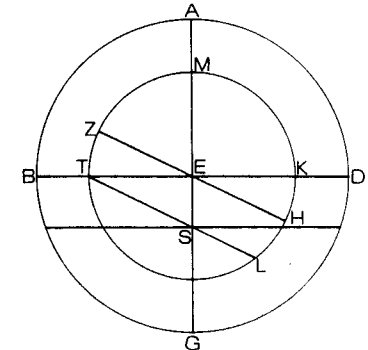



Fig. 7　　　　Fig. 8

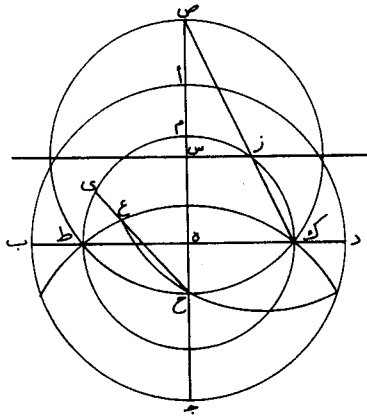
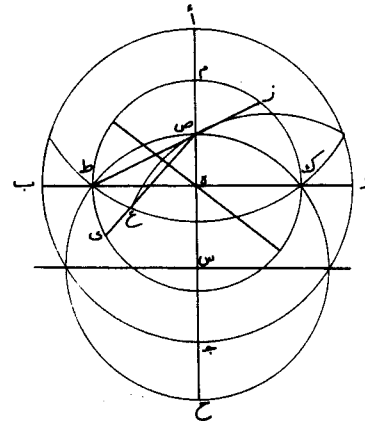
[f. 6v] *Section five*: On azimuth circles.

There are several methods to construct azimuth circles: We mentioned some of them in that book[16], while we will now use the method of Abū Maḥmūd al-Khujandī[17], on account of its simplicity and ease. We redraw the plate with the circles of the horizon and that of Aries. We take an arc *MZ* equivalent to the terrestrial latitude. In the first diagram (Fig. 9) we join *TZ*, thus

[16] Reference is made here to the *Istī'āb*, where such methods are mentioned under the section named: Projection of azimuth circles (fol. 216).

[17] For al-Khujandī, see [14], p. 74, No. 173.

القطر على صَ. وفي الثانية نصل كـ زص. فيكون صَ سمت الرأس. وندير على نقط كـ ص ط دائرة كـ ص ط ح على مركز سَ. فيكون حَ سمت الرجل. ونجيز[٧٩] على سَ خطاً مستقيماً موازياً لخط المشرق والمغرب. فعليه تقع مراكز دوائر السموت. ثم نأخذ قوس طي بقدر بعد الدائرة المطلوبة عن مطلع الاّعتدال. ونصل في الشمالي ص ي وفي الجنوبي ح ي، فيقطع[٨٠] الأفق على عَ دائرة. تسمى هذه النقطة[٨١] نقطة المجاز[٨٢]. [ب ٧ و] فنخط من[٨٣] تلك الدائرة ما وقع منها فوق الأفق ونترك سائرها. وإن كان بعد الدائرة مأخوذاً من مغرب الاّعتدال، أخذنا ذلك البعد من نقطة كَ إلى جهة مَ

شكل ١٠ شكل ٩

[٧٩] نجيز: نحيز في ب. – [٨٠] فيقطع: يقطع في ب، وهي غير منقوطة. – [٨١] عَ دائرة. تسمى هذه النقطة: عَ، وتسمى هذه المقنطرة في ت. – [٨٢] ت: ثم نطلب على خط المراكز مركزاً إذا أدرنا عليه دائرة مرت على كل واحد من سمتي الرأس والرجل وعلى نقطة المجاز. والمقصود أن ص ي يقطع الأفق على عَ، وبنقطة عَ يتم تشكيل دائرة تمر بـ: صَ، حَ، عَ. وقد أوضح البيروني طريقة الخجندي في كتاب الاّستيعاب حيث يقول: «فتكون [ع] مجاز تلك الدائرة على الأفق، فنطلب على خط مراكز السموت مركز دائرة تمر على نقطة [ع]، ونقطتي سمت الرأس والرجل فيكون ما طلبنا.»

[٨٣] فنخط من: ونحط في، في ت.

cutting the diameter at *C*. In the second one (Fig. 10) we join *KZC*. *C* would therefore be the zenith. We draw around center *S*, and through points *KCT*, the circle *KCTH*. *H* would then be the nadir. We also draw through *S* a straight line parallel to the east-west line. On that line the centers of the azimuth circles would fall. We then take an arc *TI* equivalent to the distance of the required circle from the point of ascension of the equinox. We join *CI* in the northerly astrolabe, and *HI* in the southerly one. (Line *CI*) will cut the horizon at point *O*, (which in turn will determine) a circle (with *C* and *H*). This point is called the point of intersection **[f. 7r]**. Of that circle we draw the portion that falls above the horizon and disregard the rest. If the distance of the circle (i. e. the azimuth) is measured from the setting point of the equinox, we measure that distance from point *K* in the direc-

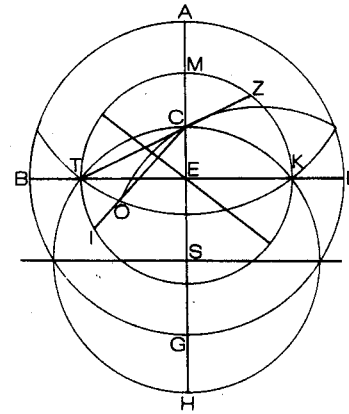
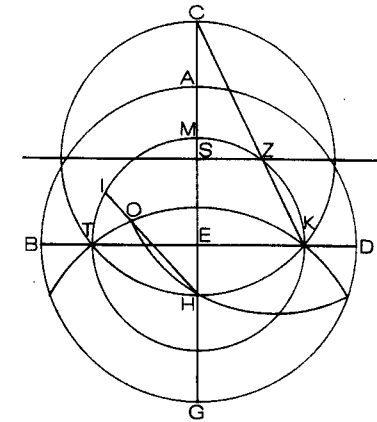



Fig. 9 Fig. 10

بدل قوس طي، وعملنا في[٨٤] العمل على ما ذكرنا. فيتم تخطيط دوائر[٨٥] السموت في نصف دائرة جميعها. وذلك ما أردنا أن نبين، والله أعلم[٨٦].

القول السادس: في قسمة المنطقة[٨٧].

[ب ٧ ظ] قد قدمنا في عمل الآفاق أنا متى أخذنا بدل عرض المسكن تمام الميل الأعظم تشكل لنا بدل الأفق منطقة البروج. فإذن نتج من ذلك أن أفق المسكن[٨٨] آلذي عرضه مساوٍ لتمام الميل الأعظم، وهو ستة وستون جزءاً وربع وسدس، مطابق لفلك البروج في بعض أوضاعه. فإذا خططنا المنطقة في العنكبوت وسمينا الصفيحة كما تقدم، على أن تفاضل أبعاد دوائر السموت عن مطلع الاعتدال ومغربه ثلاثين ثلاثين، آنقسمت المنطقة بالأبراج. وإن عملنا التفاضل جزءاً آنقسمت بدرجة درجة. وإن عملناه ستة[٨٩] آنقسمت لأصطرلاب سدس. وكذلك ثلاثة لثلث، واثنان لنصف، وعلى هذا المثال. ثم نبتدي من عند وسط السماء، إلى جهة اليسار من الأبراج المتّسِعة[٩٠]. إن كان الأصطرلاب جنوبياً فبالسرطان ونتليه بالأسد ثم العذراء ثم الميزان [ب ٨ و] ثم العقرب ثم الرامي ثم الجدي ثم الدلو ثم السمكة ثم الحمل ثم الثور ثم التوأمين. وإن كان شمالياً فبالجدي ثم ما يتلوه من البروج على النظم المذكور. فإذا فعلنا ذلك لم يبق علينا من أعمال العنكبوت إلا إظهار مواضع رؤوس الكواكب[٩١].

[٨٤] في: باقي في ت. – [٨٥] ما ذكرنا فيتم تخطيط دوائر: سقطت من ت. – [٨٦] والله أعلم: سقطت من ت. – [٨٧] ت: في العنكبوت. – [٨٨] المسكن: المسكون في ت. – [٨٩] ت: أجزاء. – [٩٠] المتسعة: التسعة في ت. – [٩١] ت: الثابتة.

tion of point *M*, instead of arc *TI*, and we proceed with the construction as before. Thus the construction of all azimuth circles in a semicircle is completed. That is what we wanted to show, and God knows best.

Section six: On the division of the ecliptic.

[f. 7v] We have already mentioned in the construction of the horizon circles that when we take instead of the latitude of a locality the complement of the maximum inclination, the construction would then produce the ecliptic, instead of the horizon. Therefore the horizon circle of a locality whose latitude is equal to the complement of the maximum inclination – being sixty-six degrees and a quarter and a sixth (of a degree) – would coincide with the ecliptic in some of its positions. So if we project the ecliptic onto the rete, and if we call the plate as we did before, such that the difference between the successive azimuth circles measured from the ascension or the setting of the equinox, is taken in thirty-degree divisions, then the ecliptic would be divided into zodiacal signs. And if we take the differences to be one (degree) at a time, (the ecliptic) would then be divided into degrees. If (the differences) were taken for six (degrees), then we would have a "sextile" astrolabe. And similarly for a three (degree difference) it would be a "trine" (astrolabe), and for the two (degrees difference), (the resulting azimuth circles) would be half (of those drawn for one degree), and so on. Then we start (the markings) to the left of the point of midheaven, which is taken in the direction of the wide signs. If the astrolabe is southerly, we start with Cancer, and follow that by Leo, Virgo, Libra **[f. 8r]**, Scorpio, Sagittarius, Capricorn, Aquarius, Pisces, Aries, Taurus, and Gemini. But if the astrolabe is northerly, (we start) with Capricorn, and then follow it with the signs in the above-mentioned order. Once we have completed that, the only thing we would still have to construct in the rete would be the positions of the pointers for the stars.

القول السابع: في رؤوس الكواكب[٩٢].

ولا بد من أن تكون مواضع الكواكب لنا معلومة مصححة بما وجد من مسيرها إلى الوقت المفروض. فإن كان ذلك حاصلاً وأردنا مواضع رؤوسها في العنكبوت فإنا نعيد مدار الحمل وهو م ط ک على مركز ه. فإذا آستخرجنا طرفي[٩٣] قطر الأفق وهما ز ح [الشكل ١١] - على أن يكون العرض[٩٤] المفروض، وهو كل واحد من ط ز، ح ک مساوٍ[٩٥] لتمام الميل الأعظم - وأدرنا الأفق، كان بعينه منطقة البروج وهي ل ط س ک. ونخرج ل س في جهة س إلى ما آمتد إليه. ونأخذ [ب ٨ ظ] م ا مثل تمام الميل الأعظم. ونصل ط ص ا. ونطلب على خط ل ه س مركز دائرة تجوز على نقط ط ص ک، فكأنه ف. والدائرة تقطع م ه س على ج فتكون نقطتي[٩٦] ص، ج سمتي الرأس والرجل، وف مجاز خط المراكز، فنجيزه عليها موازياً ل ک ه ط. ثم نأخذ بعد درج[٩٧] الكوكب من أول الحمل إن كانت في البروج الشمالية، ومن أول الميزان إن كانت في البروج الجنوبية. ونعد مثل ذلك البعد من لدن نقطة ک في النصف الأسفل من مدار الحمل[٩٨]، فكأنا آنتهينا إلى ب. ونصل ص ب يقطع المنطقة على ع وهو المجاز. ثم نطلب على خط المراكز مركز[٩٩] دائرة تمر على نقط ص ع ج، آلتي هي المجاز وسمت الرأس والرجل. ونمدها غير مؤثرة في جميع الصفيحة ونسميها دائرة العرض. ثم ننظر،

[٩٢] ت: الثابتة. – [٩٣] طرفي: طرفا في ت. – [٩٤] العرض: العروض في ت.
[٩٥] مساوٍ: مساوياً في ت. – [٩٦] نقطتي: نقطتا في ت. – [٩٧] درج: درجة في ت.
[٩٨] من مدار الحمل: صححت في هامش ب وكانت مسقطة. – [٩٩] مركز: سقطت في ب وأخذنا بالنص في ت للمعنى.

Section seven: On the pointers for the stars.

The positions of the stars should be known to us, (including) the corrections (added) to their motions up to the given time. Once that is obtained, then if we wish to find the positions of their pointers in the rete, we would then construct the day circle of Aries, which is *MTK* around center *E*. And once we determine *Z* and *H*, the two extremities of the horizon's diameter (Fig. 11) – such that the assumed latitude (of that horizon circle), marked as *TZ* and *HK*, is equal to the complement of the maximum declination – and then we construct the horizon circle. Then (that circle), *LTSK*, would be the ecliptic itself. We extend *LS* as much as possible in the direction of *S*. We measure **[f. 8v]** arc *MA* equal to the complement of the maximum declination. We join *TCA* and seek on the line *LES* the center of the circle that passes through points *T*, *C*, and *K*. Let this point be *F*. This circle would then intersect *MES* at point *G*, and the two points *C* and *G* would thus be the zenith and the nadir. *F* would be the (fixed point) through which the line of centers passes. We therefore draw (that line) through (*F*), and parallel to *KET*. Then we measure the angular distance of the star from the beginning of Aries if it is in the northerly signs, and from the beginning of Libra if it is in the southerly ones. We then measure such a distance from the point *K* in the lower[18] half of the Aries day circle, and assume we reach a point *B*. We join *CB* and let it intersect the ecliptic at point *O*, which is the common point (between the azimuth circle and the horizon)[19]. Then we seek along the line of centers the center of a circle that passes through the points *C*, *O*, and *G*, which are the points of intersection of the zenith and the nadir (respectively). We draw it lightly along the whole plate, and call it the latitude circle. We then check if the astrolabe is

[18] Lower is understood to signify zodiacal signs, which are usually taken in the upper part of the astrolabe. However, if it is taken in the other direction, i. e. the lower part of the astrolabe, the same construction would still apply.

[19] In the *Istī'āb*, (fol. 219), Bīrūnī explains this construction more clearly. He says: "... Then we measure in the day circle of Aries an arc [*KB*] equal to the distance of the required zenith from the midday line. We join [*CBO*] to cut the horizon at point *O*. Point *O* would thus be the intersection between that circle and the horizon. So we seek on the center line, the center of the circle that passes through *O* and the points corresponding to the zenith and nadir which is what we are looking for."

فإن كان الأصطرلاب شمالياً وعرض الكواكب في الشمال [ب ٩ و] أخذنا من كل واحدٍ من طرفي قطر[١٠٠] الأفق الذين هما زح مقدار عرض الكوكب إلى الجهة آلتي إليها[١٠١] تؤخذ مقنطرات الآرتفاع. وإن كان عرضه في الجنوب فمنهما إلى الجهة آلتي إليها تؤخذ مقنطرات الآنحطاط. ونخط تلك المقنطرة فحيث قاطعت دائرة العرض فثم رأس ذلك الكوكب.

وإن كان الأصطرلاب جنوبياً كان الوصل بين كاص بدل[١٠٢] طصا في الصورة الأولى [الشكل ١٢]، وكانت نقطة ص آلتي هي سمت الرأس خارج الصفيحة. وسائر الأعمال على حالها بعد تغيير الوضع في الصورة الأولى إلى الثانية، حتى نستخرج دائرة العرض. ثم إن كان عرض الكوكب جنوبياً عد مثله[١٠٣] من[١٠٤] كل، واحدة من نقطتي ح، ز[١٠٥] إلى الجهة آلتي فيها خط المراكز، وآستخرجت[١٠٦] تلك المقنطرة الآرتفاعية. وإن كان العرض شمالياً عد مثله إلى خلاف [ب ٩ ظ] تلك الجهة وآستخرجت له المقنطرة الآنحطاطية. فحيث قطعت دائرة العرض فثم رأس الكوكب، والسلام، ولله أعلم[١٠٧].

[١٠٠] قطر: قطري في ت. – [١٠١] إليها: سقطت في ب. – [١٠٢] بدل: وب دل في ب وهي مخالفة لمضمون النص. – [١٠٣] مثله: بمثله في ت. – [١٠٤] ب: خط، وأخذنا بالنص في ت لموافقة المعنى. – [١٠٥] ح، ز: ز، ح في ت. – [١٠٦] وآستخرجت: فآستخرج في ت.
[١٠٧] والسلام والله أعلم: وهو المطلوب في ت.

northerly, and the latitude of the stars to the north, [**f. 9r**] then we measure from each of the two extremities of the horizon diameter, Z and H, the value of the latitude of the stars in the direction to which latitude almucantars are taken. If, however, its (i. e. the star's) latitude is to the south, we measure from each of the extremities in the direction in which depression almucantars are taken. We then draw that almucantar, and wherever it intersects the latitude circle, we would have the pointer for that star.

If, on the other hand, the astrolabe were southerly, then we would connect KAC instead of TCA in the first diagram (Fig. 12), and point C, which is the zenith, would fall outside the plate. The remaining constructions to determine the latitude circle, would then be the same after (taking into consideration) the change in position between diagrams one and two. Then, if the latitude of the star is southerly, we measure its value from each of the two points H and Z, in the direction of the line of centers, and thus obtain that latitude almucantar. If, on the other hand, the latitude were northerly, we measure its value in the opposite [**f. 9v**] direction, and thus obtain for it the depression almucantar. Wherever it (i. e. the almucantar) intersects the latitude circle, there would the pointer for the star be. So, and God knows best.

شكل ١١

شكل ١٢

القول الثامن : في الساعات[١٠٨].

وأما تخطيط خطوط الساعات فهو أن يقسم ما تحت الأفق من مدار الآعتدالين[١٠٩]، وكل واحد من مداري المنقلبين [ب ١٠ و] بآثني عشر قسماً مستوية.

[١٠٨] في الساعات : في تخطيط الساعات الزمنية في ت. – [١٠٩] الآعتدالين : الآعتدال في ت.

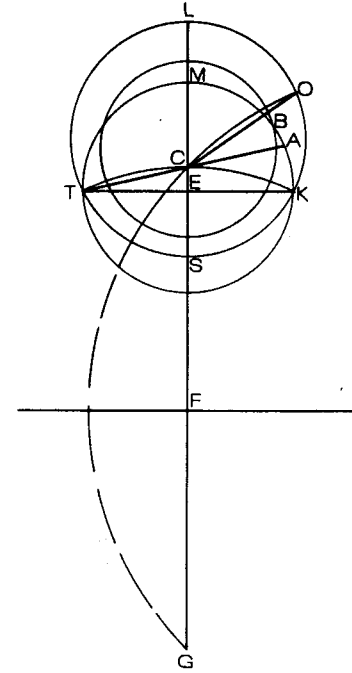



Fig. 11

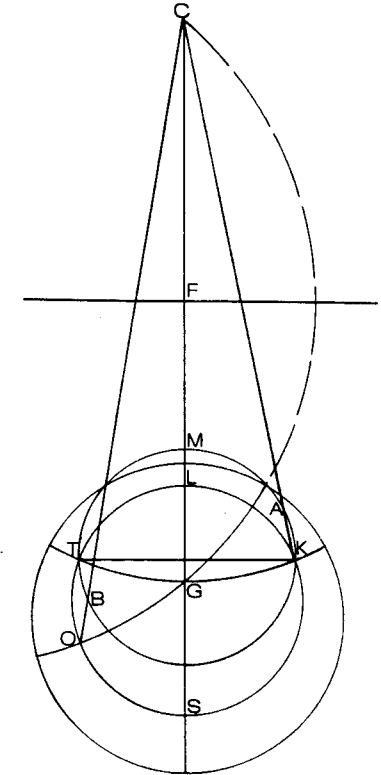



Fig. 12

Section eight: On the hours.

As for constructing the hour lines, that is (achieved) by dividing the portion under the horizon of the day circles of the equinoxes, and each of the two solstices, **[f. 10r]** into twelve portions. We then join the extremities of the corresponding arcs of

ويدار على نهاية نظيره[١١٠] في غيره من المدارين قوس من دائرة بين مداري المنقلبين، فيكون خطوط الساعات. ويتم[١١١] بها[١١٢] عمل الأصطرلاب[١١٣].

السؤال الثاني: وهو ستة أقاويل.

الأصطرلاب[١١٤]، وإن حاكى[١١٥] الأفق في حركاته، وطابقه في أشكاله وهيئته، وأمكن به الوصول إلى أكثر ما يُحسب بالزيجات، فإن أعظم فوائده ٱلتي لا عوض منها معرفة الطالع في عاجل الحال على البديهة، من غير نصب[١١٦] آلات وتسوية رخامات وتطويل قياسات وحسابات. وإذا كان كذلك، وكانت الأصطرلابات[١١٧] كبيرة كما ذكرناها، أوما ٱستُحسن منها في ذلك الكتاب[١١٨]، فلا أقل من أن نضيف ٱستخراج الطالع بكل واحد منها.

القول الأول: في ٱستخراج الطالع بنوعي الأصطرلاب[١١٩] المستعمل.

نجعل الآرتفاع بالعضادة على ظهر الأصطرلاب[١٢٠]، ثم توضع درجة الشمس في

[١١٠] نهاية نظيره: نهاية كل قسم منها وعلى نهاية نظيره في ت. — [١١١] يتم: تتم في ب.

[١١٢] بها: بتمامها في ت.

[١١٣] هنا تنتهي مخطوطة (ب) بما يلي: تمت هذه الرسالة بحمد الله وعونه وحسن توفيقه. والحمد لله رب العالمين، وصلّى الله على محمد وآله وصحبه وسلم تسليماً كثيراً. كتبت بتاريخ خامس عشر صفر سنة ستة وأربعين وسبعماية هجرية على صاحبها أفضل الصلاة والسلام. كتبها العبد الفقير إلى الله تعالى أحمد الصوفي غفر الله له ولوالديه ولجميع المسلمين. ووردت على غير هذا الترتيب في المخطوطة، وبخطين مختلفين كما يلي:
+ تمت هذه الرسالة بحمد الله، وصلى الله على محمد وآله وصحبه وسلم تسليماً كثيرا، وعونه وحسن توفيقه، كتبت بتاريخ خامس عشر صفر سنة ستة وأربعين وسبعماية هجرية، على صاحبها أفضل الصلاة والسلام، والحمد لله رب العالمين، كتبها العبد الفقير إلى الله تعالى أحمد الصوفي، غفر الله له ولوالديه ولجميع المسلمين.
+ وتسمتر المخطوطتان (ت) و(م) في نقاش السؤال الثاني من المخطوطة.

[١١٤] الأصطرلاب: الأسطرلاب في (م). — [١١٥] حاكى: حال في (م). — [١١٦] نصب: تضرب في (م). — [١١٧] الأصطرلابات: الأسطرلاب في (م). — [١١٨] ذلك الكتاب، يعني كتاب الآستيعاب. — [١١٩] الأصطرلاب: الأسطرلاب في (م). — [١٢٠] الأصطرلاب: الأسطرلاب في (م).

the day circles by a circular arc (limited) by the two day circles of the solstices. We thus obtain the hour lines. And with this the projection of the astrolabe is completed[20].

THE SECOND QUESTION: and it consists of six sections.

Although the astrolabe corresponds to the movements of the horizon and coincides with it in its configurations and appearance and (although) it is possible to obtain from it most of (the information) that can be calculated from astronomical tables, however, its greatest usage, that can not be substituted for, is in finding the horoscope in a short time and intuitively, without needing the erection of instruments and the leveling of sundials and excessive measurements and calculations. And if this is so, and the astrolabes (to be used) were big ones, like the ones we have mentioned, or what is found to be appropriate out of them (i. e. the kinds of astrolabes,) then the least that should be done is to add (a summary of the methods of) finding the horoscope by using any of them.

Section one: Concerning the determination of the horoscope by using either one of the two commonly used astrolabes.

We measure the altitude (of the sun for that day) using the alidade located on the back of the astrolabe, then we place the degree of the sun in the ecliptic on the almucantar which corre-

[20] MS. Seld. Sup. 85 ends here with the following colophon: This treatise was completed with the praise of God, His assistance, and His good fortune. Praise be to God, the Lord of all creation, and may God's prayers be upon Muḥammad and upon all his family and companions, and may He endow them with abundant peace. (This treatise) was completed (i. e. copied) on the fifteenth of Ṣafar of the year seven hundred and forty-six of the Hijra (= A. D. 17 June, 1345), may the best of prayers and peace be upon its undertaker (i. e. the Prophet). This was copied by Aḥmad al-Ṣūfī, the slave who is in need of God Almighty. May God forgive him, his parents, and all the Muslims.

المنطقة[١٢١] على المقنطرة الموافقة في العدد للآرتفاع الموجود من الجهة آلتي وقع القياس فيها في جهتي المشرق والمغرب، ثم ننظر إلى الأفق من جهة المشرق، أي برج وأي درجة وافقه[١٢٢]، فما كان فهو الطالع من فلك البروج في ذلك الوقت.

القول الثاني: في تقدير الأصطرلابات[١٢٣] الموافقة في آستخراج الطالع الذي تقدم ذكره.

ويوافق ما ذكرناه[١٢٤] في العمل بالأصطرلابات[١٢٥] الزائلة أقطاب تسطيحها عن قطب الكرة على استقامة المحور، وهي آلتي تشكلت مقنطراتها، ودوائر سموتها، ومنطقة البروج فيها، بصنوف[١٢٦] قطوع المخروط، وكذلك يوافقه المسطح تسطيحاً اسطوانياً، والمبطح، شمالياً كان أم جنوبياً، فأما سائرها فيخالفه في ذلك.

القول الثالث: في آستخراج الطالع بالأصطرلاب[١٢٧] الآسي والمطبّل.

المقنطرات المخطوطة فيهما[١٢٨] هي شمالية وجنوبية بآشتراك الأفق، فليسم ذلك الأفق مشتركاً، وليسم الأفق الآخر المعترض على المقنطرات[١٢٩] الشمالية مفرداً، ثم نقول في ذلك أنا نحصّل الآرتفاع ودرجة الشمس في المنطقة، فإن كانت شمالية الميل وضعناها[١٣٠] على مثل ذلك الآرتفاع في المقنطرات الشمالية، وفي جهة القياس. وينظر[١٣١] إلى الأفق المشترك[١٣٢] من جهة اليسار، فإن وافقه برج شمالي[١٣٣] فهو الطالع

[١٢١] المنطقة: المنطقم في (م). – [١٢٢] وافقه: واقعه في (م). – [١٢٣] الأصطرلابات: الأسطرلابات في (م). – [١٢٤] ذكرناه: ذلاناه في (م). – [١٢٥] الأصطرلابات: الأسطرلابات في (م). – [١٢٦] بصنوف: يصفون في (م). – [١٢٧] الأصطرلاب: الأسطرلاب في (م). – [١٢٨] فيهما: منهما في (م). – [١٢٩] المقنطرات: المسطرات في (م).
[١٣٠] وضعناها: وصفناها في (م). – [١٣١] وينظر: تنظر في (م). – [١٣٢] المشترك: سقطت في (م). – [١٣٣] شمالي: شمال في (م).

sponds in number to the altitude found on the side in which the measurement has been taken, i. e. in either east or west directions. Then we look at the eastern side of the horizon (and see) which degree in which zodiacal sign corresponds to it. Whatever is found, that would be the ascending point of the ecliptic (i. e. the horoscope) at that time.

Section two: On approximating (other) astrolabes, (where the method of finding the horoscope corresponds) to the (method) previously mentioned.

What we have mentioned applies when working with astrolabes where the pole of projection does not lie on the pole of the sphere along the axis. These are (the astrolabes) whose almucantars, azimuth circles, and ecliptic are projected into the kinds of conic projections. Moreover, it applies to those that are projected by cylindrical projections, and to the northerly and the southerly planispheric (astrolabes). As for other kinds of astrolabes, they differ in their usage.

Section three: On finding the horoscope by using the myrtle-like and the drum-like astrolabes.

The almucantars that are constructed in those two (kinds) are northerly and southerly, (having) a common horizon. Let us call that horizon the common horizon, and let us call the horizon which intersects the almucantars the singular one; we thus say concerning those (kinds) that we find the altitude (of the sun for that time of the day,) and the position of the sun in the ecliptic. If it (i. e. the sun) has a northerly declination, then we place it (i. e. the point corresponding to the sun on the ecliptic) at the corresponding altitude in the northerly almucantars, and on the side on which the measurement has been taken. We then look at the common horizon from the left side (i. e. on the eastern horizon, and see): If it intersects (the ecliptic in a point which falls in) a northerly zodiacal sign, then that degree and minute would

بدرجته ودقيقته، وإن وافقه برج جنوبي نظرنا الأفق المفرد في جهة اليسار، فما وافقه فهو الطالع بدرجته. وإن كانت درجة الشمس جنوبية الميل قلب الأصطرلاب[١٣٤] حتى تصير المقنطرات الجنوبية إلى فوق والشمالية إلى تحت، ووضع الدرجة على مثل الآرتفاع المحصّل في المقنطرات الجنوبية وفي جهة القياس[١٣٥]، ثم[١٣٦] ينظر إلى الأفق المشترك من جهة اليسار، فإن وافقه برج جنوبي فهو الطالع بدرجته ودقيقته، وإن وافقه برج شمالي ترك ونظر إلى المفرد، فيؤخذ الطالع بدرجته موافقاً إياه.

[١٣٤] الأصطرلاب: الأسطرلاب في (م) و (ت). — [١٣٥] القياس: غير مقروءة في (م).
[١٣٦] ثم: م في (م).

be the horoscope. And if it intersects a southerly zodiacal sign, then we look at the singular horizon from the left side. The corresponding (point) would be the degree of the horoscope. If, however, the position of the sun (in the ecliptic) has a southerly inclination, then we invert the astrolabe so that the southerly almucantars are upwards and the northerly ones are downwards. We put the point (corresponding to the sun in the ecliptic) on the determined altitude in the southerly almucantars, and on the side on which the measurement has been taken. Then we look at the common horizon from the left side. If it intersects (the ecliptic at a point which falls in) a southerly zodiacal sign, then this would be the minutes and degree of the horoscope. If, on the other hand, it intersects a northerly zodiacal sign, then we look at the singular horizon, and the degree of the horoscope would then be the same as the corresponding (intersection) point (between the singular horizon and the ecliptic.)

Fig. 13

Fig. 14

القول الرابع: في آستخراج الطالع بالأصطرلاب[١٣٧] الزورقي.

يوضع النصف الشرقي من الأفق المجسم على درجة الشمس من المنطقة، ويعلم على موضعها منه علامة الدرجة، وعلى موقع[١٣٨] العمود من الحجرة علامة. ثم توضع علامة الدرجة على مثل الآرتفاع المقيس في جهة المقياس. وينظر كم بين[١٣٩] خط وسط السماء ورأس العمود في الحجرة، فما[١٤٠] كان فهو الدائر من الفلك من وقت طلوع الشمس. ثم يعد من علامة موقع العمود الأول من الحجرة إلى جهة اليسار مثل الدائر، + وتوضع على رأس العمود على منتهاه +[١٤١]، وينظر إلى الأفق الشرقي المجسم[١٤٢] أي[١٤٣] جزء وافق من أجزاء البروج، فهو الطالع.

القول الخامس: في آستخراج الطالع بالأصطرلاب[١٤٤] الصليبي والمسطري[١٤٥]

ينظر[١٤٦] إلى درجة الشمس في الصليب، فإن كانت شمالية الميل، وضعت على الأفق من جهة المشرق وعلم على موقع أحد الأطراف الماسة للحجرة، ثم وضعت تلك

[١٣٧] الأصطرلاب: الأسطرلاب في (م) و (ت). – [١٣٨] موقع، موضع في (م). – [١٣٩] بين: هي في (م). – [١٤٠] فما: كما في (م). – [١٤١] +...+: والأفضل أن نقرأ: يوضع رأس العمود على منتهاه، أي منتهى الدائر. – [١٤٢] الشرقي المجسم: سقطت في (م). – [١٤٣] أي: آلتي في (م).
[١٤٤] بالأصطرلاب: بأصطرلاب في (ت). – [١٤٥] والمسطري: للسطري في (م).
[١٤٦] ينظر: قنطر في (م).

Section four: On finding the horoscope by using the boat-like astrolabe.

We place the eastern half of the solid horizon at (the point corresponding to) the position of the sun in the ecliptic, and we mark on the horizon the corresponding point, and also mark the position of the pointer[21] (the outer rim of) the mater. We then place the mark (corresponding to the) position (of the sun on the ecliptic) at the (almucantar circle) corresponding to the measured altitude and on the side on which the measurement is taken. We then measure (the arc) between the line of midheaven and the head of the pointer along the mater; the resultant would then correspond to the rotation of the ecliptic since sunrise. We then count (back) in the direction of the east from the first pointer mark on the mater, an (arc) equal to the rotation of the ecliptic, and we place the tip of the pointer at the extremity of that (angle). The horoscope would then be that point of the ecliptic that intersects the point on the eastern solid horizon.

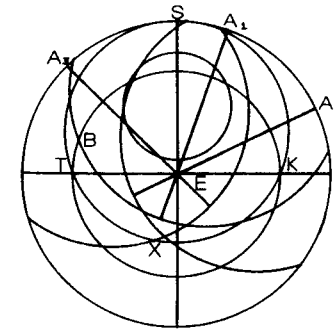



Fig. 15

Section five: On finding the horoscope by using the cross-like and the straight edge astrolabes.

We look for the position of the sun in the cross(-like rete). If it has a northerly inclination, we place it on the eastern horizon, and we mark one of the edges intersecting the mater. We then

[21] The word pointer we are using here translates Arabic *'āmūd* which literally means the 'post', i. e. the mast of the ship.

الدرجة على مثل الآرتفاع الموجود في المقنطرات الشمالية، وفي جهته. فما زال ذلك الطرف[١٤٧] فهو الدائر من الفلك. وإن كانت درجة الشمس جنوبية الميل، قلب الأصطرلاب[١٤٨] حتى تصير المقنطرات الجنوبية فوق، ثم وضعت على الأفق من جهة[١٤٩] اليسار، وعلم على موقع أحد الأطراف من[١٥٠] الحجرة. ثم وضعت على مثل الآرتفاع المقيس من المقنطرات الجنوبية وفي جهته. فما زال ذلك الطرف فهو الدائر من الفلك. فإذا حصل الدائر عد[١٥١] مثله على توالي البروج من درجة الشمس في مطالع البلد المقسومة في جزء من الصفيحة. فحيث آنتهى العاد فهو الطالع بدرجته. وكذلك القول في المسطري، إلا أن مقنطراته نوع واحد والعمل بها أيضاً لا يتنوع.

[١٤٧] ذلك الطرف: الفلك بالطرف في (م). – [١٤٨] الأصطرلاب: الأسطرلاب في (م).
[١٤٩] جهة: سقطت في (م). – [١٥٠] أحد الأطراف من: الحد الأطراب في (م). – [١٥١] عد: عبر في (م).

place that point on the corresponding altitude in the northerly almucantars, and on its side (i. e. on the side in which the altitude measurement was taken.) The (angular) displacement (of the point marked on the mater) would thus be the rotation of the ecliptic. If on the other hand, the position of the sun has a southerly inclination, then we invert the astrolabe so that the southerly almucantars would be in the upper part. We then place (the point corresponding to the position of the sun) on the eastern horizon, and we mark one of the edges intersecting the mater. We place that point on the corresponding measured altitude in the southerly almucantars, and on its side, (i. e. on the side on which the measurement was taken.) The (angular) displacement of that edge would thus be the rotation of the ecliptic. Once we obtain (the amount of) rotation, we count an arc equal to it, in the direction of the order of the signs, starting from the position of the ascension of the sun which is marked on the plate. Wherever the counting ends, that would be the degree of the horoscope. The same description applies to the straightedge (astrolabe,) except that it has only one kind of almucantar and it can not be used alternatively.

Fig. 16

القول السادس: في ٱستخراج الطالع بالأصطرلاب اللولبي[١٥٢].

توضع درجة الشمس على أفق المشرق ويعلم على موقع رأس الجدي من أجزاء الحجرة. ثم[١٥٣] توضع درجتها أيضاً على مثل الآرتفاع الموجود وفي جهته، فما زال المري عن موضعه فهو الدائر من الفلك، فنعمل به حينئذ ما تقدم ذكره في الصليبي والمسطري[١٥٤]، حتى يخرج الطالع. وفضلية اللولبي[١٥٥] على غيره هي إمكان ٱنقسام البروج فيه[١٥٦] بالدرج والدقائق مع وضع قدر الأصطرلاب.

فهذه أصناف الأصطرلابات ٱلتي ذكرناها في ذلك الكتاب، ومولاي يكفيني من الكثير بالقليل، ويكفي بذكائه وفهمه[١٥٧] مؤنة الكثير الطويل. والله يقيه[١٥٨] ومن الأسواء يعينه[١٥٩]، ويرثني السعادات المأمولة فيه.

[١٥٢] اللولبي: الكوكبي في (م). — [١٥٣] الحجرة. ثم: الحجر تم في (م). — [١٥٤] والمسطري: السطري في (م). — [١٥٥] وفضيلة اللولبي: وفصيلته في (م). — [١٥٦] فيه: فإنه في (م).

[١٥٧] بذكائه وفهمه: بذكاء فهمته في (م). — [١٥٨] يقيه: غير مقروءة في (م). — [١٥٩] يعينه: غير مقروءة في (م).

Section six: On finding the horoscope by using a spiral astrolabe.

We place the (point corresponding to) the position of the sun (on the ecliptic) on the eastern horizon, and we mark on the mater the position of the beginning of Capricorn (which is on the spiral rete). Then we also place its position (i. e. the point corresponding to the position of the sun on the ecliptic) on the (almucantar) corresponding to the determined altitude (of the sun) and on its side (i. e. on the side on which the measurement was taken.) The angular displacement of the pointer (i. e. the tip of Capricorn) from its location would thus be the rotation of the ecliptic. We then proceed with it as we proceeded with in the case of the cross-like and straightedge astrolabes, until we obtain the horoscope. The virtue of the spiral (astrolabe) over other kinds is that its zodiacal signs can be divided into degrees and minutes, while using the same size of astrolabe.

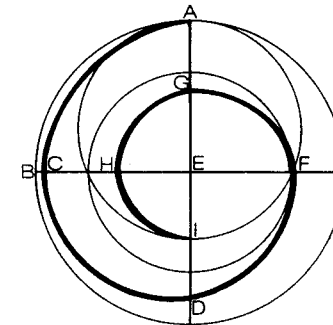



Fig. 17

These are the kinds of astrolabes that we had mentioned in that book (i. e. *al-Istī'āb*), and this brief (account) would suffice to provide my master for the excess (of others,) and his sharp mind and understanding would save me from (going into) details. May God support him and protect him from harm, and may He grant me the expected beneficence from him.

وتم الكتاب بالعون[١٦٠] من الله المعين. هذا كتاب أبي الريحان محمد بن أحمد البيروني[١٦١] إلى[١٦٢] أبي سهل عيسى بن يحي المسيحي الطبري[١٦٣] في تسهيل التسطيح الأصطرلابي، والعمل بالمركبات الشمالي والجنوبي منه. وهو سؤالان وأربعة عشر قولاً.

فرغ من آنتساخه أحوج خلق[١٦٤] الله مهران بن أمير الحاج القيصري في عاشر رجب الأصب + . . . +[١٦٥]، والله خير موفق.

[١٦٠] العون: لمن في (ت). – [١٦١] البيروني: النردى في (م). – [١٦٢] إلى: ابن في (م).
[١٦٣] إلى أبي سهل عيسى المسيحي الطبري: سقطت وأضيفت على الهامش في (ب).
[١٦٤] آنتساخه أحوج خلق: الساحة أجمع لخلق في (م). – [١٦٥] + ... +: غير مقروءة في (ت) وسقطت في (م).

The book was completed with the help of God.

This is the book of Abū al-Rayḥān Muḥammad b. Aḥmad al-Bīrūnī to Abū Sahl ʿĪsā b. Yaḥyā al-Masīḥī al-Ṭabarī on simplifying astrolabic projections, and on working with their northerly and southerly combinations. It is composed of two questions and fourteen sections.

The most needy of God's creations, Mihrān b. Amīr al-Ḥājj al-Qayṣarī finished transcribing it on the tenth of the Only Rajab, . . ., and God is the Best Helper.

Commentary

THE FIRST "QUESTION":

Section one: In this section, Bīrūnī tries to list the different kinds of conic sections that result from cutting a circular cone by a plane. These vary as follows:

If in a cone a plane is drawn to cut a section, such that the cutting plane makes an angle α with the axis of the cone, then the resulting intersection is a function of the relationship between α and β, which is the generating angle of the cone.

Now if we are projecting circles in the sphere on the equatorial plane, then this is equivalent to having a cone cut by a fixed plane. However the cone itself would vary in this case according to the position of the pole of projection, which is the vertex of the cone, and to the position of the base of the cone, which is the circle to be projected. The fixed cutting plane is that of the equator. Again α and β would vary simultaneously, and different conic sections would be obtained as follows:

1. The pole of projection is on the axis inside the sphere.

a. If the base circle passes through the pole of projection, then the cone is reduced to a circle, and the intersection between the cone and the equator will be an intersection between two planes, thus a straight line.

b. If the base circle of the cone is parallel to the equator, then $\alpha = 90^0$, and the section cut is a circle.

c. If now we assume any fixed angle rotating around the pole of projection, then this is equivalent to having this angle fixed and rotating the cutting plane, then we have the following possibilities:

- $\beta < \alpha < 90$ $\Rightarrow$ the intersection is an ellipse.
- $\alpha = \beta$ $\Rightarrow$ the section is a parabola.
- $\alpha < \beta$ $\Rightarrow$ the section is a hyperbola.

Thus when the pole of projection is on the axis inside the sphere, the projection admits all kinds of possible conic sections: Straight lines, circles, ellipses, parabolas, and hyperbolas.

2. The pole of projection on the axis outside the sphere.

Here also the cone admits all kinds of sections, i. e. straight lines, circles, ellipses, parabolas, and hyperbolas. This can easily be shown in a fashion similar to the preceeding case. However, Bīrūnī mentions only the possibilities of straight lines, circles and hyperbolas; yet in his *Istī'āb*, he maintains that all sections may be produced.

3. The pole of projection on the axis on the pole of the sphere. Only two possibilities arise in this case:

a. The circle to be projected passes through the pole of projection, and the section is a straight line.

b. The circle to be projected does not pass through the pole; in this case the cutting section, (i. e. the equator,) is either parallel to the circle to be projected, or is a subcontrary section. In either case the projection would be a circle, which is a property of stereographic projections. (For an elaborate work on projections of spheres, other than stereographic projections, see [10].)

In this section, Bīrūnī also lists the different kinds of circles represented on the plate of the astrolabe. On it appear families of curves that represent the maps of corresponding families on the celestial sphere. These, as is clearly stated by Bīrūnī, are: Horizon circles, azimuth circles, day circles, and almucantars. (On the kinds of circles represented in the astrolabe, see Hartner's essay on the *asṭurlāb* in [6].)

Section two: On the construction of the day circles.

Bīrūnī's terminology at this point needs some clarification. He seems to be operating for construction purposes with a circle that is just used as a drawing board called the *dastūr*. In the celestial sphere the plane of the *dastūr* circle coincides with the plane of the meridian.

The plane of the astrolabic projection, however, coincides with the plane of the equator, which appears in Figure 1 as the circle with center E and radius EM. In this circle the north pole of the celestial sphere falls above E, and the meridian or *dastūr* plane appears as a single line along AEC. In order to facilitate the construction, the meridian plane is rotated around MC through 90^0, so that the north pole arrives at K, and the meridian circle arrives at circle $TMKC$. Now a point H on the meridian is mapped into the point S on the equator plane ME; but when the meridian plane is folded back to regain its position in space, S remains where it is, since it falls on the axis of rotation.

In the above construction, Bīrūnī proceeds as follows:

a. Take any circle $ABGD$ of any desired dimension.

b. Measure arc AZ equal to the inclination of the ecliptic, ε. Join BZ, and let it intersect AEG at M.

c. Draw circle MTK around center E, and let this circle represent the projection of the equator.

Now in Figure 1 we have taken arc AZ to be equal to ε. Therefore DZ is equal to the complement of ε. Therefore angle $OBT = (90^0 - \text{arc } OT)/2$, being an exterior angle to circle $MTCK$. But angle $OBT = (90^0 - \varepsilon)/2$, being inscribed in circle $ABGD$. And since triangles AET and BME are symmetrical, therefore arc FM = arc $OT = \varepsilon$.

So when taking circle $MTCK$ to correspond to the meridian plane, point F would be the beginning of Capricorn. Hence A would be the projection of F. Therefore circle $BADG$ would be the projection of the tropic of Capricorn.

Section three: On the horizon circles.

In this section Bīrūnī begins by taking the line of intersection between the horizon and the meridian plane. When the meridian plane is rotated down into the equatorial plane, this line of intersection will assume the position ZH in Figure 3. He then projects it onto the equatorial plane. Since it is one of the diameters of the horizon, its projection on the equator will determine the projection of the circle of the horizon. Let that line be LO.

When the meridian plane is folded back to fall along the line AEG, the circles are preserved in a stereographic projection. Thus the projection of the horizon circle is a circle with diameter LO. Furthermore, this circle must pass through T and K. The reason for this is that the equator intersects with any horizon at the east and west points, T and K, which map into themselves, because they are on the equator.

In the projection of the ecliptic, on the other hand, Bīrūnī follows the same method by assuming the ecliptic itself to be a horizon of latitude equal to ε. At that point the same conditions used for the horizon will be applicable to the ecliptic.

Section four: On the almucantars.

In this section Bīrūnī discusses the projection of almucantar circles, which are also called altitude or depression circles. They are the small circles of the celestial sphere which are parallel to the horizon and at different altitudes. These circles will project as chords on the meridian plane TMK (Fig. 5). The points L and O will be projected into the points S and F in the equatorial plane. When the two planes are rotated back, S and F retain their positions, being located on the axis of rotation. Now since circles are mapped into circles, then the almucantar circle LO will be projected into a corresponding circle on the equatorial plane, having SF as a diameter.

A special case occurs when the depression of a depression almucantar is the negative of the latitude of the locality, as illustrated in Figures 7 and 8. For then the almucantar passes through the pole of projection, and the point of intersection maps at infinity. However, the other extremity of the required diameter is entirely normal. It is L in both figures and its map is S. The map of the given circle of depression must pass through S at right angles to the map of the meridian. Hence it is the straight line OSF for both figures 7 and 8. (The letters O and F have been omitted from the latter.)

Section five: On azimuth circles.

The syntax of the Arabic text of this section is difficult to understand (see note). What it seems to be saying is the following: To construct the azimuth circle we first construct the equator and the horizon circles (Fig. 9). On the meridian plane we mark MZ equal to the latitude of the locality in question. Point Z corresponds to the zenith of this locality. We then join TZ, where T is the pole of projection for a northerly astrolabe,to cut diameter ME at C. C would thus be the projection of the zenith of the locality.

Now the prime vertical is the circle perpendicular to the horizon through the zenith, and which has a zero azimuth; hence it passes through the east and west points, through T, K and C. The resulting circle $TCKH$ cuts line CS at H, which is the map of the nadir.

Since all azimuth circles pass through both zenith and nadir, then their projections pass through C and H, and the set of centers of those projections is the perpendicular bisector of CH, or in other words, the line drawn through S, which is the center of the circle $KCTH$, parallel to the east-west line.

Now for any given azimuth circle, the azimuth arc is measured on the horizon circle. We do know that in stereographic projec-

tions, circles are projected into circles, and angles are preserved. However, the arc lengths of the projected circles are not preserved. Therefore, in projecting the azimuth circle, the value of the azimuth arc cannot be measured directly on the horizon circle. We can apply several methods for the above projection. Hartner mentions one in his article on the astrolabe. The method used here is different, however, and it assumes previous trigonometric computations. An arc *TI* is measured along the equator, where arcs are preserved, such that this arc represents the distance of the required circle from the "point of ascension of the equinox", the east point on the horizon, *I*. We join *CI* to cut the horizon circle at *O*. Circle *COH* would thus be the required azimuth circle.

It is evident in the above construction that Bīrūnī is assuming a knowledge of the values of arcs on the equator, which correspond to values of arcs on the horizon, as marked by azimuth circles. This, we know was calculated and probably tabulated at the time of Bīrūnī, and indeed he refers to methods for calculating these arcs in his *Istī'āb* (fol. 214–218).

Section six: On the division of the ecliptic.

The ecliptic, together with its related coordinate system of celestial latitudes and longitudes can be regarded as a set of horizon coordinates for a locality where the local latitude is 90^0-ε, as Bīrūnī remarks. This will be the case provided the rete is rotated so that the point on it which maps the first point of Aries coincides with the east point on the map of the local horizon on the plate. The ecliptic is represented by a ring on the rete. The points on the edge of the ring which make up the graduations marking ecliptic degrees may be found by laying out the azimuth circles for the special latitude just mentioned.

If the azimuth circles are taken six degrees apart, then the resulting number of azimuth circles is equal to a sixth of the total drawn if there were one for each degree. Bīrūnī calls the former kind a "*sudus*", sextile astrolabe. Similarly, if the difference between the azimuth circles is either three degrees or two degrees, then the number of azimuth circles would be a third or a half of the total possible.

In the above construction different signs on the ecliptic are projected in different sizes on the rete. Bīrūnī calls wide signs

those like Sagittarius and Capricorn, whose projections on the rete are bigger arcs than the projections of, say, Gemini or Cancer.

Section seven: On the pointers for the stars.

If the rete is placed in the special position described in the preceding section, then the known latitudes and longitudes of the astrolabe fixed star may be regarded as altitudes and azimuths of a locality of terrestrial latitude 90^0-ε. By the technique used to lay out the coordinate net for horizon coordinates, the caftsman draws a circle which is the map of the circle of latitude (almucantar) of a particular star. A second circle maps the longitude (azimuth) of the same star. The intersection of the two circles fixes the point on the rete for the pointer of that particular star.

THE SECOND "QUESTION":

Having given a detailed description of the method of construction of the two astrolabes of standard type Bīrūnī then proceeds to solve the practical problem of horoscope determination. The solution of this problem, however, is given not only for the common type of astrolabe, but also for a variety of other kinds. The construction of these astrolabes is summarily discussed in Bīrūnī's long treatise *al-Istī'āb*.

Section one: Horoscope determination for the standard variety of astrolabe.

To find the horoscope in a general planispheric astrolabe, which is the kind discussed in the first question of the treatise, we first take the altitude of the sun at the time for which the horoscope is to be cast, by using the alidade as a sighting instrument. We then locate the almucantar which corresponds to that altitude. Now, assuming that the position of the sun on the ecliptic for that part of the year is known, rotate the rete until this position falls on the specified almucantar, then seek the point on the ecliptic that is rising over the eastern horizon; that point will be the horoscope.

Section two: Horoscope determination for those astrolabes where the above discussed method holds.

Bīrūnī claims, without proof, that the above method applies

also for astrolabes which are constructed by using all kinds of conic projections, and whose poles of projection do not coincide with either the north or the south poles of the celestial sphere. He also claims that astrolabes constructed by cylindrical projections, can be used in the same manner. We refrain here from commenting because we are not sure of the method and the construction that Bīrūnī had in mind for this case.

Section three: Horoscope determination using the myrtle-like and the drum-like astrolabes.

In his *Istī'āb*, Bīrūnī describes the construction of the above two astrolabes. In both kinds the almucantar circles are drawn for the northern and the southern portions of the astrolabe. Bīrūnī calls the horizon that limits the northern section "the common horizon", while the one that limits the northerly section, and thus intersects the northern almucantars, is called singular. The ecliptic for both a northerly and a southerly astrolabe is also drawn. In the myrtle-like astrolabe (Fig. 13) we mark that portion of the ecliptic corresponding to the northern zodiacal signs in the ecliptic projected for a northerly astrolabe, and erase the remaining part of that circle. We thus keep the portion below the east-west line *TK*. Similarly, of the southerly ecliptic, we mark only that portion corresponding to southern zodiacal signs, i. e. the portion falling above the east-west line. Once these two parts are formed, we rotate the two ecliptics through ninety degrees, and the resulting rete will have the shape of a myrtle. (For further details see the *Istī'āb*, fol. 254–255.)

In the drum-like astrolabe (Fig. 14) the southerly zodiacal signs are marked out of the signs of the northerly signs in a southerly ecliptic. The resulting rete (as in the figure) is bulged, and hence the name drum-like astrolabe. (For further details see the *Istī'āb*, fol. 253–254.)

In the above two kinds of astrolabes we proceed to find the horoscope as follows:

First we find the altitude of the sun for that time of the day, and the position of the sun for that time of the year. If the sun has a northerly inclination, we move the rete to the side in which the measurement is taken, until the point on the rete corresponding to the sun falls on the northerly almucantar corresponding to the measured altitude. We then look at the point of the ecliptic rising above the eastern side of the common horizon. If this point falls in a northerly zodiacal sign, then it is itself the horoscope. But if it falls in a southerly zodiacal sign, then the horoscope would be the point of the ecliptic rising above the eastern side of the singular horizon.

Section four: Horoscope determination using the boat-like astrolabe.

The construction of this astrolabe is based on a theoretical assumption that requires the celestial sphere to be fixed, and the earth to rotate within that sphere. For an observer on the earth, the celestial phenomena would still look the same as they did when the earth was taken to be fixed, and the celestial sphere in motion. Bīrūnī attributes this construction in the *Istī'āb* (fol. 267–269) to Abū Sa'īd al-Sijzī, and correctly observes that for mathematical astronomers it does not make any difference as to which of the two is taken as a fixed reference point. The computation would still be the same, since one only measures relative motions. He further asserts that such discussions should be relegated to the natural philosophers. As far as he was concerned, he was satisfied with the fact that the astrolabe resulting from such an assumption could still determine the horoscope.

Therefore, for this construction we take a fixed ecliptic, and allow the horizon to move. A movable pointer (a solid bar) is also attached to the horizon along the line of midheaven (Fig. 15), and it can also serve as an alidade.

To determine the horoscope with this astrolabe, we first locate the position of the sun on the ecliptic for that time of the year. We then move the eastern part of the solid-body horizon to intersect the ecliptic at the above point. Let point A_1 be the intersection between the pointer tip and the outer rim, and mark on the horizon point B, which is the intersection between the ecliptic and the solid-body horizon.

We then rotate the horizon so that point B falls on the altitude almucantar corresponding to the measured altitude of the sun for a specific time of the day, and on the same meridian side on which the measurement was taken. The pointer will now point towards point A_2 on the outer rim. The arc SA_2 between the midheaven line and the pointer EA_2 corresponds to the rotation of the ecliptic since sunrise, that is, it corresponds to the actual ro-

tation of the sun between sunrise and that time of the day. This rotation, however, should be measured back from point A_1, which was the position of the tip of the pointer when the sun was taken to be at B. We thus measure from A_1, to the east side, an arc $A_1A_3 = SA_2$, and rotate the solid-body horizon so that the pointer tip falls on A_3. Point X, the intersection between the horizon and the ecliptic, would thus be the required horoscope.

Section five: Horoscope determination using the cross-like and straight edge astrolabes.

In the present text of Bīrūnī, the functioning of the cross-like astrolabe is essentially the same as that of the boat-like astrolabe mentioned above. He first determines the rising point of the sun marked at the mater, and then determines the amount of the daily rotation of the sun as an arc along the same mater. Finally he takes that daily rotation arc in a backward direction from the rising point. The point on the ecliptic corresponding to the last mark on the mater would then be the horoscope.

The straightedge astrolabe is constructed by taking the following projection of the ecliptic. Assume the meridian line on a regular astrolabe to be a solid fixed ruler. Then with a compass open for a radius equal to ET (Fig. 16), where E is the projection of the north pole, and T the point at the beginning of Aries, and with one side of it fixed at E, mark with the other side point T_1, along the meridian line that corresponds to T. Note that for the beginning of Aries T_1 falls along the equatorial circle. Repeat the same procedure for the beginnings of the other zodiacal signs, and mark their corresponding points along the straightedge meridian line. The resulting rete would therefore be a straightedge along the meridian line. (For further details on the construction of this astrolabe see *Istī'āb* fol. 270–271.)

The construction of the cross-like astrolabe, on the other hand, is very briefly stated in the *Istī'āb* (fol. 271–272). The intention of the cross-like astrolabe seems to have been, according to Bīrūnī's statement, an attempt by al-Sijzī, to improve the straightedge astrolabe by allowing larger scale divisions for the solistial signs which are normally compressed in the straightedge as in the figure. At this point, and with the available sources, the exact construction of this astrolabe is not very clear. We hope that a more detailed description of its workings will be unearthed in the future.

Section six: Horoscope determination using a spiral astrolabe.

The construction of this astrolabe is as follows: On an appropriate plate draw the day circles for the beginnings of each of the zodiacal signs. Then starting with the outer circle mark point A on one edge of the quadrant AEB, and point C on the next inner circle in the same quadrant. Join AC with a curve, and with the same procedure connect, with similar curves, points D, F, G, H, and I. The resulting curve, the rete, will look like a spiral, with the sections in each quadrant approximating the day circles of two of the zodiacal signs. Section AC will correspond to the day circle of Sagittarius and Capricorn, CD to Scorpio and Aquarius and so on. (For further details see *Istī'āb*, fol. 272–273.)

The horoscope is determined, as before, by locating the point corresponding to the sun on the rete. We move that point so that it falls on the eastern horizon, and we mark the corresponding intersection between the pointer, which is the tip of Capricorn, and the outer rim. We then move the same point so that it falls on the almucantar corresponding to the sun's altitude for that day. Again we mark the intersection between the pointer and the outer rim. We measure the arc corresponding to the movement of the pointer along the outer rim. This would be the rotation of the ecliptic since sunrise. Now we proceed with the construction as we did in the earlier cases, namely we rotate the first point marked on the outer rim by an arc equal to the rotation of the ecliptic since sunrise, and in the eastern direction. The point where the rete intersects the eastern horizon would thus be the horoscope.

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